Globalization and Wage Polarization

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Abstract

The US labour market has experienced a remarkable polarization in the 1980s and 1990s. At the same time, the US faced a fast technological catch-up as European countries and especially Japan drastically improved their global innovation and patenting activity. Is foreign technological convergence an important source of the recent evolution of the US wage and employment structure? To answer these questions, we set up a Schumpeterian model of endogenous technological progress with two asymmetric countries, heterogeneous workers, endogenous skill formation and occupational choice. Calibrating the model to match key facts of the US economy, we find that foreign technological catching-up observed between the late 1970s and early 1990s reproduces a non-negligible part of US wage polarization. Moreover, the model delivers predictions on the US wealth to income ratio consistent with empirical evidence in that period.

JEL Classification: F16, J31, O33
Keywords: wage polarization, heterogeneous workers, wealth-income ratio, endogenous technical change, international technology competition, personal service sector.

1 Introduction

The US labour market has experienced a radical polarization of employment and wages in the last decades. In the 1980s and 1990s, both wages and employment shares at the tails of the skill distribution have grown steadily. Workers in the middle of the distribution instead have faced stagnant wages and a shrinking employment share (Acemoglu and Autor, 2011). Recent empirical work\(^1\) has shown that also the wealth to income ratio has been increasing steadily since the mid-1970s in the US. Here, we make an explorative attempt to assess the role of globalization, in the form of fiercer foreign technological competition, in shaping these important developments of the United States economy.

In the years of expanding labour market polarization, the US economy became increasingly globalized. The massive reduction in trade barriers and the diffusion of technologies across

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\(^1\) See for example Piketty (2014) and Piketty and Zucman (2014).
countries’ borders allowed foreign firms, mostly from Japan and Europe, to challenge US technological leadership. The geography of technological leadership, measured as countries’ share of innovation inputs and outputs, shows remarkable changes between the mid-1970s and the late 1980s. As the distribution of leadership moved from drastically skewed toward the US, to a more equal global playing field, clear convergence patterns can be observed in the share of patents, patent citations and R&D spending. The share of foreign patents in the US Patent Office, was about one third in 1977 and grew to about one half ten years later. The US share of global industrial R&D declined from about 50 percent in 1979 to 39 percent in 1995. Most of this technological catching-up was due to a massive acceleration in Japanese innovation activity, although some European countries, such as Germany and France, played a major role in some sectors (Impullitti, 2010, and Akcigit, Ates, and Impullitti, 2014).

Did the acceleration of foreign technological competition in the 1980s contribute to the polarization of the US labour market? To answer this question we construct a quality ladder growth model (Aghion and Howitt, 1992, and Grossman and Helpman, 1991) with two asymmetric countries and heterogeneous workers. Firms compete for global market leadership investing in quality-improving innovation. Schumpeterian competition for innovation allows successful innovators to replace incumbents. The asymmetry between countries is represented by a technology gap in innovation: firms in the leading country have a better innovation technology in all sectors of the economy. There are three occupations: innovation, production, and services: innovation workers are employed in the production of new ideas to improve the quality of the goods produced by their firms. Blue collar, production workers are employed in the manufacturing of goods. Service sector workers provide personal services that allow their employer to save working time. Workers have heterogeneous ability and they can acquire working skills through education. Educational attainment allows workers to become skilled and work in innovation activities. Workers who do not acquire education can work as unskilled production workers or in service occupations.

In equilibrium, the following allocation of abilities to occupations is obtained: workers with high innate ability become skilled and hire service sector workers, those with intermediate ability work in production occupations and finally, those at the bottom of the ability distribution work in personal service occupations. An exogenous reduction in the technology gap affects the leading economy’s innovation and labour market structure as follows. First, as foreign firms start innovating more efficiently, they obtain quality leadership in more sectors, thereby stealing market shares from firms in the leading country and reducing the demand of production workers. This business-stealing effect reduces wages of production workers in the leading economy. Second, the reduction in the technology gap makes it harder for domestic firms to innovate in the global economy, thus pushing them to devote more labour resources to innovation. This global competition effect triggers an increase in the demand for skilled workers. Via these two channels, increasing foreign competition generates more polarization in the leading economy’s labour market: the wage of skilled workers relative to that of production workers
increases, thereby raising inequality at the top of the skill distribution. The wage of service sector workers relative to that of production workers increases, thus reducing inequality at the bottom of the distribution. The increase in the demand and wages of service sector workers is a by-product of rising inequality at the top of the distribution. As skilled labour time becomes more valuable, skilled workers demand more personal services in order to free time to devote to their highly remunerative job.

Another key prediction of our model is the positive link between foreign technological competition, innovation, and the wealth to income ratio. The business-stealing and the global competition effects of foreign technological catching up increase the value of innovation in the global economy. The value of innovation in our framework is determined by the value of the leading firm in each product line, and aggregate wealth consists in the sum of the market values of all leading firms. Hence, our model establishes a natural connection between international competition, innovation and the dynamics of the wealth to income ratio.

In an extended version of the model, we allow for a more general technology where skilled and unskilled workers are employed in production and innovation with different factor intensities, and we introduce iceberg trade costs. We then calibrate this generalized model and use it to explore the link between the technology gap, labour market polarization and the wealth to income ratio quantitatively. In our two-country world, the US is the leading economy and the rest of the world represents the foreign country. The technology gap between countries is calibrated to reproduce the distribution of patents in the US Patent Office observed in the data. Our quantitative exercise involves reducing the innovation technology gap to reproduce the convergence in US and foreign patent shares shown in the data between 1980 and 2000. We then compare the increase in polarization triggered by this reduction in the technology gap with that observed in the data, and find that our model can replicate about 13% of the increase in inequality at the top of the distribution, 26% of the increase in the skill premium, and about 23% of the decrease in inequality at the bottom observed between 1980 and 2000. Finally, the decline in the technology gap generates a striking increase in the US wealth to income ratio, reproducing a substantial part of the increase during this period documented by Piketty and Zucman (2014).

The recent empirical evidence on labour market polarization in the US has triggered a new line of research aimed at understanding and interpreting these facts. Autor and Dorn (2013) present a simple “task-based” model with capital-skill complementarity formalizing the so-called "routinization" hypothesis. This hypothesis posits that technological change complements skilled tasks, replaces routine tasks, and is neutral to service occupations. Hence technology affects the wage structure through a postulated factor bias. We propose a complementary approach according to which polarization is driven by the factor intensity of innovation rather than the factor bias of technological progress. Moreover, Autor and Dorn treat tech-

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2The capital skill-complementarity idea was first introduced by Krusell, Ohanian, Rios-Rull and Violante (2000) to study the evolution of the US skill premium in a two-skill model.
nological progress and the supply of skills as exogenous. We complement their analysis by endogenizing both technological change and skill formation, as well as by exploring the key role of globalization.

In our economy, the increase in the demand and wages of service sector workers relative to other unskilled tasks comes from a general equilibrium market mechanism: the increase in innovation triggered by foreign competition raises inequality at the top of the distribution, thereby inducing skilled workers to demand more personal services and devote more time to their highly paid jobs. This mechanism finds direct empirical support in Mazzolari and Ragusa (2013) who, using US city-level data, show that the increase in the top wages bill can explain about one-third of the growth of employment of non-college workers in low-skill personal services in the 1990s.

Our paper is also related to the literature focusing on globalization as a source wage inequality. A large body of work has studied the effects of trade liberalization on wage inequality across workers with different skills when technology is constant (e.g. Yeaple, 2005, Epifani and Gancia, 2008, Burnstein and Vogel, 2010), or when technology is endogenous and interacts with trade in shaping the wage structure (e.g. Dinopoulos and Segerstrom, 1999, Acemoglu, 2003, Thoenig and Verdier, 2003). We depart from this literature along two main lines: first, while existing papers focus on economies with two skills and explain the evolution of the skilled-unskilled wage gap, the skill premium, we set up a framework with a continuum of skills allocated to three occupations to study the dynamics of wage inequality in the middle and in the tails of the distribution. Second, we move from a widely studied dimension of globalization, trade liberalization, to the less explored channel of cross-country technological catch up. A small emerging literature has started to analyse the welfare effects on leading economies of foreign technological catching up (e.g. Eaton and Kortum, 2007, Hsie and Ossa, 2011, Impullitti, 2010, Akcigit, Ates, and Impullitti, 2014). Our paper contributes to this line of research studying the effects of foreign catching up on the structure of wages, employment, and on the evolution of the wealth to income ratio.

Our paper is organized as follows. Section 2 shows the stylized facts that motivate the paper. Section 3 sets up our baseline model, Section 4 characterizes its steady state equilibrium and Section 5 analytically derives its comparative statics properties. Section 6 is devoted to extending the model to match empirical regularities of the US economy, which is then used for a quantitative assessment of our theory. Section 7 concludes.

2 Stylized facts

In this section, we discuss some key facts providing the motivation for the paper as well as the empirical support for the quantitative analysis.
**Wage polarization.** Since the early 1980s, the US has shown a remarkable increase in labor market polarization. The distribution of wages and employment shares across workers with different skill levels has become more unequal in the period between 1980 and the present. Autor and Dorn (2013) document a non-monotonic change in employment and wages along the skill distribution. Working with Census IPUMS and American Community Survey data, they rank 318 occupations in all US nonfarm employment by skill level using the average log wages of workers in each occupation. Their results show that employment changes in the period 1980-2005 have an inverted U-shaped pattern. Employment in the middle of the skill distribution declines substantially, while the tails show a steady and somewhat puzzling increase. Digging deeper into the occupational structure of these changes they document that most of the increase in employment and wages in the lower tail can be attributed to one group of occupations that they name service occupations. These are low-education occupations involving caring, assisting and entertaining other people, such as, cleaners, janitors, security guards, food service workers, gardeners, home health aides, hairdressers, beauticians, and recreation occupations. Table 1 below shows the employment dynamics for three groups of occupations. Skilled workers include the occupations at the top of the skill distribution. Unskilled workers outside service occupations comprises a low-educated occupational group including production and crafts jobs, operative and assembly occupations, transportation, construction, mechanical, mining, clerical and retails sales jobs. The third group includes the service occupations described above. The data show a strong increase in the employment share of skilled workers, which grew by 25% between 1980 and 2000. Similarly, the employment share of service occupations increased by about 17% in the 1980-2000 period. On the other hand, the employment share of unskilled non-service occupations declined by about 16%.

<table>
<thead>
<tr>
<th>Levels</th>
<th>growth</th>
<th>1980</th>
<th>1990</th>
<th>2000</th>
<th>80-00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skilled workers</td>
<td>31.55</td>
<td>38.16</td>
<td>39.61</td>
<td></td>
<td>0.25</td>
</tr>
<tr>
<td>Manager/Prof/Tech/Finance/Public Safety</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unskill no-service</td>
<td>58.52</td>
<td>51.31</td>
<td>48.77</td>
<td></td>
<td>-0.16</td>
</tr>
<tr>
<td>Production/Craft</td>
<td>4.82</td>
<td>4.78</td>
<td>3.54</td>
<td></td>
<td>-0.23</td>
</tr>
<tr>
<td>Transp/Constr/Mech/Mining/Farm</td>
<td>21.5</td>
<td>18.7</td>
<td>18</td>
<td></td>
<td>-0.16</td>
</tr>
<tr>
<td>Machine Operators/Assembly</td>
<td>9.94</td>
<td>7.30</td>
<td>5.70</td>
<td></td>
<td>-0.42</td>
</tr>
<tr>
<td>Clerical/RetailSales</td>
<td>22.2</td>
<td>21.6</td>
<td>21.4</td>
<td></td>
<td>-0.04</td>
</tr>
<tr>
<td>Unskill service</td>
<td>9.91</td>
<td>10.51</td>
<td>11.61</td>
<td></td>
<td>0.17</td>
</tr>
</tbody>
</table>

Source: Autor and Dorn (2013)

Table 2 shows the levels and changes in wages for the three groups of occupations in the same period. We can see a substantial increase in wages of skilled workers and a smaller but non-negligible increase in service occupations’ wages. The wages of unskilled workers in non-service occupations instead exhibit virtually no change in the last decades.
Table 2. Mean Real Hourly Wages by Major Occupation Groups: 1980-2000

<table>
<thead>
<tr>
<th>Levels</th>
<th>1980</th>
<th>1990</th>
<th>2000</th>
<th>growth 80-00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Log Hourly Wage (2004$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Skilled workers</td>
<td>2.82</td>
<td>2.89</td>
<td>3.02</td>
<td>0.19</td>
</tr>
<tr>
<td>Unskill no-service</td>
<td>2.58</td>
<td>2.58</td>
<td>2.62</td>
<td>0.04</td>
</tr>
<tr>
<td>Unskill service</td>
<td>2.09</td>
<td>2.14</td>
<td>2.24</td>
<td>0.14</td>
</tr>
<tr>
<td>Skill Premium</td>
<td>1.37</td>
<td>1.47</td>
<td>1.60</td>
<td>0.17</td>
</tr>
<tr>
<td>Skill/Unskill(no serv)</td>
<td>1.27</td>
<td>1.36</td>
<td>1.49</td>
<td>0.21</td>
</tr>
<tr>
<td>Unskill(no serv)/Serv</td>
<td>1.62</td>
<td>1.55</td>
<td>1.46</td>
<td>-0.16</td>
</tr>
</tbody>
</table>

Source: Autor and Dorn (2013)

Focusing on the relative changes across these groups, we can look at the dynamics of the gap between the top and the middle of the distribution and that between the middle and the bottom. The first gap is represented by the ratio of skilled over unskilled (non-service) wages, which shows a 21% increase up to 2000. The second gap is the ratio between the wages of unskilled workers outside of personal service occupations and those of service workers. This ratio declines by 16 percentage points up to 2000. We also compute a more standard measure of inequality, the skill premium, defined as the average wage of skilled workers over of the average wage of all unskilled workers, including service sectors workers. In line with inequality at the top of the distribution, the skill premium shows a remarkable increase after 1980.

The wealth to income ratio. Piketty and Zucman (2014) report that the wealth to income ratio in the US rose from slightly above 350% in 1980 to 450% in 2000. The wealth of a nation is defined as the sum of domestic capital and net foreign assets. In the US, the foreign asset position in those years is negligible, hence the wealth to income ratio and domestic capital to income ratio coincide. For this reason, they use the terms wealth-income and capital-income ratio interchangeably. Capital in the data is the sum of agricultural land, housing, the value of financial assets net of liabilities and the value of net non-financial assets. Figure 2 shows the evolution of private wealth and its components in the period we are analysing. As we can see, financial wealth is the driving force of the increasing wealth to income ratio, while housing wealth (net of mortgages) and non-financial assets, which include tangible capital, are fairly

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3The average unskilled wage is obtained using the employment shares of the unskilled (no service) and service occupation workers a weights.
constant.\footnote{Non financial assets include produced tangible capital and non-produced tangible capital. Tangible assets is the typical economic measure of physical capital which includes all tangible fixed assets: buildings and structures, machinery and equipment, cultivated biological resources, weapon systems, etc. Non-produced tangibles includes natural resources such as land, oil, gas, etc. See Piketty and Zucman (2014) online data appendix for details.}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{Composition_of_Private_Wealth_1980-2000.png}
\caption{Composition of Private Wealth, 1980-00, \% National Income}
\end{figure}

About two thirds of financial assets are accounted for by corporate wealth. In Figure 2, we report the two measures of corporate wealth constructed by Piketty and Zucman (2014).\footnote{Piketty and Zucman (2014) online appendix table US.6b.} The first measure is the equity value, which is the market value of the firm, and the second is the net worth, which is the “book value” of the firm which comes from a perpetual inventory method-based estimates of tangible assets. The difference between the two measures is the residual corporate wealth, and their ratio is the well-known Tobin’s Q.

Source: Piketty and Zucman (2014)
Both measures of corporate wealth exhibit a striking increase in the period of interest. The book value rises from 150% of income in 1980 to 220% in 2000, while the equity value from 67% in 1980 skyrockets to 280% in 2000.

**International technological competition.** During this period of increasing wage polarization and fast growing wealth, the US experienced increasing foreign technological competition. The data show a global convergence pattern in several measures of innovation performance. The US, the undisputed technological leader in the post-World War II period, was progressively challenged by Japan and European countries in the late 1970s and 1980s. Figure 3, shows the stark convergence in patent shares in the US Patent office; in 1977 about two thirds of patents came from American firms, while ten years later only one half of patents had US origins. Most of the convergence takes place in the 1970s and 1980s, while the following years show no major changes.

This convergence in patenting activity did not happen because of a slowdown in US patenting but it is due to a stark acceleration in foreign patents. The number of foreign patents grew
by about 60% between 1977 and 1988, while US patents did not show any sensible change in those years. This increase is mainly driven by Japan, whose share rose from 9% in 1977 to about 20% in 1988. Along these lines, Akcigit, Ates, and Impullitti (2014) show that a similar trend can be seen in patent citations. Disaggregating at the level of technology classes, they document a convergence trend in global patenting also at the micro/sectorial level. Together with convergence in innovation output (patents), recent research has also highlighted similar patterns for global innovation inputs. Impullitti (2010) documents a strong convergence in private R&D spending between the US and a group of fast growing advanced economies in the 1970s and 1980s.

**Figure 3. Share of US and Foreign Patents in US Patent Office, 1977-2005**

In what follows we build a theory to analyse the effects of foreign technological competition on the wage and occupational structure, and on the dynamics of the wealth to income ratio. We then use the data discussed here to quantitatively assess the link between foreign competition and our key measures of inequality. More precisely, we use the model to replicate the change in patent shares observed in Figure 3, measure the changes in wage polarization and the wealth to income ratio implied by the model and compare them to those in the data shown in Table
The model

In this section, we set up a Schumpeterian growth model of a global economy with two countries separated by a technology gap in their innovation efficiency, and trading at no extra costs. There are three occupations, production of goods, innovation, and personal service. Workers have heterogeneous abilities, acquire skills through education and sort across different occupations. We use the model to analyse the interaction between globalization, innovation and the occupation and wage structure.

3.1 Households

Consumption. The economy is populated by two regions with the same population and preferences. In both regions, households are heterogeneous in their abilities to acquire skills. At birth, members draw their ability \( \theta \in [0,1] \) from a distribution \( \Gamma(\theta) \). Households have identical preferences for a continuum of consumption goods \( \omega \in [0,1] \), and each is endowed with a unit of labour or study time whose supply generates no disutility. Household of type \( \theta \) is modelled as dynastic family that maximizes intertemporal utility

\[
U = \int_0^\infty N(0)e^{-(\rho-n)t} \ln u_\theta(t) dt,
\]

where population is specified according to \( N(t) = N(0)e^{nt} \), with initial population \( N(0) \) normalized to 1 and a constant population growth rate \( n \). The rate of time preference is \( \rho \), with \( \rho > n \). The utility per person is given by

\[
u_\theta(t) \equiv \left[ \int_0^1 \left( \sum_{j=0}^{j_{\max}(\omega,t)} \lambda^j d_\theta(j,\omega,t) \right)^{\frac{n-1}{\eta}} d\omega \right]^{\frac{\eta}{\eta-1}},
\]

where \( d_\theta(j,\omega,t) \) is the per member quantity of good \( \omega \in [0,1] \) of quality \( j \in \{0,1,2,...\} \) purchased by a household of ability \( \theta \) at time \( t \geq 0 \). A new vintage of good \( \omega \) yields a quality \( \lambda \) times that of the previous vintage, with \( \lambda > 1 \). Different versions of the same good \( \omega \) are regarded by consumers as perfect substitutes after adjusting for their quality ratios, and \( j_{\max}(\omega,t) \) denotes the maximum quality in which the good \( \omega \) is available at time \( t \). Parameter \( \eta \in [1,\infty) \) is the elasticity of substitution across varieties, which allows for gross substitutability across varieties.

At each point in time, households choose the quantity purchased of each good \( d_\theta(j,\omega,t) \) in order to maximize (2) subject to the per period expenditure constraint \( \int_0^1 d_\theta(j,\omega,t)p(j,\omega,t)d\omega = c_\theta(t) \), where \( c_\theta(t) \) is planned time \( t \) consumption expenditure. Notice that the household will
be purchasing in each line only the product with the lowest price per unit of quality. Hence, household’s demand for each product is:

$$d_\theta(j, \omega, t) = c_\theta(t) q(\omega, t) P(t)^{n-1} p(j, \omega, t)^n,$$

(3)

where $q(\omega, t) = \lambda^{(n-1)} j^{\max}(\omega, t)$ represents the quality of good $\omega$ and $P(t)$ is a general price index, defined as:

$$P(t) = \left[ \int_0^1 q(\omega, t) p(j^{\max}(\omega, t), \omega, t)^{1-\eta} d\omega \right]^\frac{1}{1-\eta}.$$

Notice that the quality of the demanded version of a variety, $q(\omega, t)$, positively affects its quantity, while negatively affecting the overall price index. Given the optimal allocation of expenditures across different product lines at a given moment $t$ in (3), households choose the intertemporal allocation of consumption maximizing (1) with respect to the intertemporal budget constraint $a_\theta(t) = w_\theta(t) + (r(t) - n) a_\theta(t) - c_\theta(t)$, where asset market clearing implies that per capita assets are equal to the sum of firm values $a(t) = \int_0^1 v(\omega, t) d\omega$. This problem yields

$$\frac{\dot{c}_\theta(t)}{c_\theta(t)} = r(t) - \rho,$$

(4)

the standard Euler equation for consumption.

**Supply of skills.** Individuals are finitely-lived members of infinitely-lived households, being continuously born at rate $\beta_n$ and dying at rate $\delta$, with $\beta_n - \delta = n > 0$; $V > 0$ denotes the exogenous duration of their life$^6$. They choose to acquire education and become skilled, if at all, at the beginning of their lives; and for the duration of their schooling period, denoted $T_H < V$, the individual cannot work. We assume agents have heterogeneous innate abilities drawn from a cumulative distribution function $\Gamma(\theta)$.

In country $K = D$ (domestic), $F$ (foreign) an individual with ability $\theta$ decides to acquire education if the expected wage income is larger than that of working as unskilled, that is:

$$\int_t^{t+V} e^{-\int_t^s r(r) dr} \theta w^K_L(s) ds < \int_{t+T_H}^{t+V} e^{-\int_{t+T_H}^s r(r) dr} \max(\theta - \gamma_H, 0) w^K_H(s) ds,$$

(5)

where $w^K_L$ and $w^K_H$ are the wage per unit of abilities of the unskilled and skilled workers, respectively. Parameter $0 < \gamma_H < 1$ establishes a threshold ability requirement so that an agent with ability $\theta > \gamma_H$ is able to accumulate $\theta - \gamma_H$ units of skills after schooling, while a person with ability below $\gamma_H$ gains no skills from education. Parameter $\gamma_H$ can be interpreted as a fixed cost of education in terms of the skilled wage.$^7$ We will here focus on steady-state

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$^6$It is easy to show that the above parameters cannot be chosen independently, but that they must satisfy $\delta = n/(e^{nV} - 1)$ and $\beta = ne^{nV} / (e^{nV} - 1)$ in order for the number of births at time $t$ to match the number of deaths at $t + V$.

$^7$This way of modeling the accumulation of skills has been first introduced by Findlay and Kierzkowski (1983), and extended to heterogeneous agents by Dinopoulos and Segerstrom (1999). Unlike these models, we also feature heterogeneous abilities among unskilled workers, which is an important empirical aspect of the labour market.
analysis, in which all variables grow at constant rate and $w_L$, $w_H$, and $c_\theta$ are all constant\(^8\). From the Euler equation (4) we obtain $r(t) = \rho$ at all dates. Equation (5) yields the cutoff ability level $\theta^K_0$ above which agents acquire education:

$$\frac{w^K_H}{w^K_L} = \sigma_1 \left( \frac{\theta^K_0}{\theta^K_0 - \gamma_H} \right),$$

with $\sigma_1 \equiv \left[ \left( e^{\rho V} - 1 \right) / \left( e^{\rho (V - T_H)} - 1 \right) \right] > 1$. Notice that, since $0 < \gamma_H < 1$, we have $\partial \left( w^K_H / w^K_L \right) / \partial \theta^K_0 < 0$: an increase in the relative skilled wage reduces the ability cutoff to acquire education.

Each unskilled individual can work either in production occupations or as a personal service worker. Personal services allow people to spend less time on house chores, baby-sitting, transportation, and other activities, which would otherwise detract time from other kinds of activities, such as production and innovation occupations. Since in this model individuals do not have a desire for leisure, we can assume that each hour saved thanks to service workers will be used to work more. We will also posit that there is an upper bound, normalized to 1, of the extra work made possible by personal services.\(^9\) Moreover, each unit time of personal service provides its employer, who must be a different person, $1 - \epsilon$ extra time for work, with $0 < \epsilon < 1$. This implies that in equilibrium, unskilled workers of same or lower ability levels will not demand services, because it would cost $w^K_L$ to obtain an extra labour time which pays $w^K_L (1 - \epsilon)$.

A high skilled worker of generic ability $\theta$ finds it profitable to buy personal services from an unskilled worker of ability $\theta'$, if at the cost of $\theta' w^K_L$ they can obtain an additional wage income $w^K_L (1 - \epsilon) \theta \geq \theta' w^K_L$. The lowest possible $\theta$ at which hiring a personal service is worthwhile in country $K$, denoted $\theta^K_{HS}$, and the higher possible $\theta'$ supplied by the unskilled, denoted $\theta^K_{LS}$, must equalize the service cost and employer’s economic benefit, that is:

$$\theta^K_{LS} w^K_L = w^K_H (1 - \epsilon) \left[ \theta^K_{HS} - \gamma_H \right].$$

The mass of service workers must be equal to the mass of skilled workers employing them,

$$\Gamma(\theta^K_{LS}) = 1 - \Gamma(\theta^K_{HS}).$$

Plugging equation (8) into (7) and solving for $\theta^K_{LS}$ gives

$$\theta^K_{LS}(\theta^K_{HS}) \equiv \Gamma^{-1} \left( 1 - \Gamma(\theta^K_{HS}) \right) = \frac{w^K_H}{w^K_L} \left(1 - \epsilon \right) \left[ \theta^K_{HS} - \gamma_H \right],$$

which since $\Gamma^{-1}(\cdot)$ is monotonically increasing, gives $\theta^K_{HS}$ as a decreasing function of the relative skilled wage, and therefore $\theta^K_{LS}$ as an increasing function of the relative skilled wage. Intuitively,

\(^8\)An analysis of the transitional dynamics could be carried out numerically, along the lines of Cozzi and Impullitti (2010) and Cozzi and Galli (2014).

\(^9\)There is no restriction in this normalization, this upper bound can be easily generalized.
a higher relative remuneration of skilled occupations increases the demand for service occupations. Hence, invoking (6), we can write:

\[
\frac{d\theta_{HS}^K}{d\theta_0^K} > 0 > \frac{d\theta_{LS}^K}{d\theta_0^K}. \tag{10}
\]

The fraction of the population opting out of education determines the per capita supply of unskilled labour \(L^K \equiv \tilde{\theta}_L^K \Gamma(\theta_0^K)\), where

\[
\tilde{\theta}_L^K = \left[ \int_0^{\theta_0^K} \frac{d\Gamma(\theta)}{\Gamma(\theta_0^K)} \right] \tag{11}
\]

is the average ability of a generic unskilled worker. Notice that \(dL^K/d\theta_0^K = \theta_0^K > 0\), the supply of unskilled workers, is increasing in \(\theta_0^K\). Intuitively, a higher relative skilled wage reduces the cutoff ability to acquire education \(\theta_0^K\), thereby reducing the share of unskilled workers.

A fraction \(1 - \Gamma(\theta_0^K)\) of the population decides to attain education and the skilled workforce is represented by the subset of these agents that as of date \(t\) have completed their schooling period, that is individuals born between \(t - V\) and \(t - T_r\). The per capita supply of skilled labour in efficiency units at time \(t\) is then

\[
H^K = \tilde{\theta}_H^K [1 - \Gamma(\theta_0^K)] \int_{t-V}^{t-T_r} \beta_n N(0) e^{ns} ds = \tilde{\theta}_H^K [1 - \Gamma(\theta_0^K)] \phi, \tag{12}
\]

with \(0 < \phi \equiv (e^{n(V-T_r)} - 1) / (e^{nV} - 1) < 1\) and

\[
\tilde{\theta}_H^K = \left[ \int_{\theta_0^K}^{1} (\theta - \gamma_H) \frac{d\Gamma(\theta)}{1 - \Gamma(\theta_0^K)} \right] = (2 - \epsilon) \int_{\theta_0^K}^{1} (\theta - \gamma_H) \frac{d\Gamma(\theta)}{1 - \Gamma(\theta_0^K)} + \int_{\theta_0^K}^{\theta_0^K} (\theta - \gamma_H) \frac{d\Gamma(\theta)}{1 - \Gamma(\theta_0^K)}. \tag{13}
\]

is the average ability of educated workers, which takes into account that above the cutoff \(\theta_{HS}^K\) workers get and extra \(1 - \epsilon\) working time from hiring service workers. Using (13) and (10), we derive

\[
dH^K/d\theta_0^K = - \left[ (1 - \epsilon) \frac{d\theta_{HS}^K}{d\theta_0^K} + 1 \right] (\theta_0^K - \gamma_H) \Gamma'(\theta_0^K) \phi < 0,
\]

hence the supply of skills is decreasing in \(\theta_0^K\).

### 3.2 Production

In each country, firms can hire unskilled workers to produce consumption good, \(\omega \in [0, 1]\), under a constant returns to scale technology with one worker producing one unit of product. The unskilled wage rate is \(w_L^K\) and we set \(w_L^K = 1\), so that the unskilled foreign wage is the numeraire of this economy. Assuming instantaneous price competition, Bertrand equilibrium implies that in each industry only the product with the highest quality is produced. Quality leaders in each
sector are challenged by followers that employ skilled workers to discover the next top-quality product. In this model, as typical of quality ladder frameworks, a patent expires as soon as the next top-quality product is invented. Successful innovation yields global market leadership, which is protected by a perfectly enforceable patent law.

We assume that the technologies to produce goods one quality ladder below the top are obsolete and diffuse freely. This assumption allows foreign successful innovators to become global market leaders. We allow for international trade in goods, but we do not consider multinational companies in this model. In addition, while allowing for perfect patent protection, we rule out an international market for patent royalties. Therefore, in our model, only domestic firms can produce and export the goods patented in the country. Finally, we do not consider international trade in assets.

As will become clearer in the next sections, a consequence of our assumption of technological gap between the two countries - i.e. country $D$ is more efficient in innovation than country $F$ - country $D$ gains global market leadership in a wider range of sectors, which drives up the demand for its manufacturing labour. For this reason, in all our equilibria the unskilled wage in country $D$ will be higher than that of country $F$, that is $w^D_L > w^F_L = 1$.

Since both domestic and foreign followers operate with the same technology, and foreign unskilled labour is cheaper, as we will prove, domestic followers do not represent an effective competitive threat. Moreover, we assume that the quality jump $\lambda$ is not high enough that each top-quality producer firm can maximize profits unconstrained in both markets. Thus the price $p^K(j^{\text{max}}(\omega, t), \omega, t)$ of a top quality good in each sector $\omega$ at time $t$ is

$$p^K(j^{\text{max}}(\omega, t), \omega, t) = \lambda w^F_L(t) = \lambda > 1$$

in any country $K = D, F$. Notice that this implies that $w^D_L \leq \lambda w^F_L$, because otherwise country $D$ firms would lose all markets and unskilled jobs. This "narrow gap" case (Grossman and Helpman, 1991) allows for equilibrium product-cycle trade (Vernon, 1966) with global market leadership shifting from domestic to foreign firms as the latter innovate and viceversa. Although the foreign region has a cost advantage in production, equilibrium guarantees that the wage

---

10Which guarantees that within a country the firm profits are invariant regardless of the patent holder of the top-quality product - i.e. even if the industry leader owns (by purchase or innovation) the patent on the new product.

11Without this assumption if a leader experiences successive innovations, followers will be pushed out of the market permanently. The assumption of immediate diffusion of the old production technology is discussed in Glass (1997) and widely used both in North-North models of trade and growth (e.g. Dinopoulos and Segerstrom, 1999), and in North-South models (Glass, 1997, Glass and Saggi, 1998).

12Otherwise a domestic firm could license a top quality good patent to a foreign firm, extracting all profits. Realistically speaking, in the period we consider in the paper (1979-1995) intellectual property royalties from foreign unaffiliated firms did not play any major role for the United States: receipts never exceeded 7.5 billion US$ (Bureau of Economic Analysis, 2007, data table A6.3), hence we can safely ignore this possibility.

13Cases in which this does not hold can be easily handled, with no qualitative change in the results. We have decided not to spell them out explicitly in order for the exposition to be less cumbersome. Hence in all our calibrations constraint $\eta/(1 - \eta) > \lambda$ must hold.

14This includes those in the service sector, because innovation would stop - due to lack of expected profits in manufacturing, and skilled workers would lose their jobs as well.
gap is not so large that a foreign follower can price a domestic leader out of the market without innovating.

From the static consumer demand (3), we can conclude that the demand for each product $\omega$ is:

$$N(t)(c^D(t) + c^F(t)) \frac{q(\omega, t)}{P(t)_{\eta-1}} = d(\omega, t),$$

(15)

where $c^K(t) = \int_0^1 c^K_{\theta}(t) d\theta$ for $K = D, F$ are average per capita expenditures at time $t$. Notice that under the pricing condition (14) the price index becomes,

$$P(t)_{\eta-1} = \frac{1}{\lambda} \frac{1}{\int_0^1 q(\omega, t) d\omega} = \frac{1}{\lambda^{\eta-1} / Q(t)},$$

where $Q(t) = \int_0^1 q(\omega, t) d\omega$ is the average quality in the economy. Therefore, using (15) and dropping time indexes for notational simplicity, we can write:

$$d(\omega) = \frac{N(c^D + c^F)q(\omega)}{\lambda Q}, \text{ for } \omega \in [0, 1].$$

(16)

Since supply and demand of goods are equal in equilibrium, the stream of monopoly profits accruing to domestic quality leaders is

$$\pi^D_n(\omega) = d(\omega) \left( \lambda w^F_L - w^D_L \right) = \frac{N(c^D + c^F)q(\omega)}{\lambda Q} \left( \lambda - w^D_L \right),$$

(17)

and the profits of the foreign leaders are

$$\pi^F(\omega) = \frac{N(c^D + c^F)q(\omega)(\lambda - 1)}{\lambda Q}. $$

(18)

Notice that $\pi^F(\omega)$ is lower than the unconstrained monopoly profit $N(c^D + c^F)q(\omega) / \eta Q$, because the quality jump size is not enough to allow the firm to ignore the possibility of the competitive fringe re-entering the market at monopolistic prices. This induces the new market leader to limit-price the competitive fringe and sell their goods at a price equal to the quality-adjusted unit cost of the potential entrants in country $F$.

### 3.3 Global innovation races and the value of a firm

In each industry, firms employ skilled workers to discover the next top-quality version of their products. The arrival rate of innovation in industry $\omega$ at time $t$ is $I(\omega, t)$, which is the sum of the Poisson arrival rate of innovation produced by all firms targeting product $\omega$. The innovation technology available to a firm $i$ in region $K$ for innovation in sector $\omega$ is

$$I^K_i(\omega, t) = \frac{A^K h^K_i(\omega, t) \left( \frac{H^K(\omega, t)}{X(\omega, t)} \right)^{-\alpha}}{X(\omega, t)},$$

(19)

where $h^K_i(\omega, t)$ is labor input in innovation by firm $i$, in industry $\omega$ at time $t$, and $X(\omega, t) > 0$ measures the degree of complexity of innovation, $\alpha$ is a positive congestion parameter, and
\( H^K(\omega, t) = \sum_i h^K_i(\omega, t) \) and \( I^K(\omega, t) = \sum_i I^K_i(\omega, t) \) are the total skilled labour and the total innovation rates in sector \( \omega \) and country \( K \), respectively. This technology implies that each firm’s instantaneous probability of success is a decreasing function of the total domestic labour resources devoted to innovation in an industry. A possible interpretation of this property is that when firms increase innovation inputs in a sector, the probability of duplicative innovation effort also increases, thereby reducing the probability that any single firm will discover the next vintage of goods. Therefore, the sector-specific negative externality in innovation technology produces decreasing returns to innovation at the industry level. Moreover, (19) implies that this negative externality is also region-specific;\(^{15}\) this feature can be motivated by the presence of fixed costs, such as lab equipment, by institutional differences, and by the presence of a workforce with heterogeneous ability in research.\(^{16}\)

Notice that equation (19) implies:

\[ I^K(\omega, t) = A^K \left( \frac{H^K(\omega, t)}{X(\omega, t)} \right)^{1-\alpha}. \]  

(20)

The complexity index \( X(\omega, t) \) is introduced to avoid the counterfactual prediction of the first generation innovation-driven growth models that the size of a region affects its steady-state growth (Jones, 1995). Following Li (2003), we eliminate the strong scale effect by assuming

\[ X(\omega, t) = \frac{q(\omega, t)}{Q(t)^{\phi_1}}. \]  

(21)

with \( 0 < \phi_1 < 1 \). Therefore, the more advanced the good relative to the average quality the more difficult a further innovation. Moreover, even for an average quality good (i.e. if there exists an \( \omega \) such that \( q(\omega, t) = Q(t) \)), the higher the average quality itself the more difficult innovation (in this case \( X(\omega, t) = Q(t)^{\phi_1} \)), which incorporates Jones’ (1995), Kortum’s (1997), and Segerstrom’s (1998) increasing complexity argument to rule out the strong scale effect.\(^{17}\) Equation (21) implies that, log-differentiating the difficulty index between quality jumps in the sector, its evolution obeys \( \dot{X}(\omega, t)/X(\omega, t) = -\phi_1 \dot{Q}(t)/Q(t) \).

Each innovating firm chooses the labour resources devoted to innovation \( h^K_i \) in order to maximize its expected discounted profits. Free entry into innovation races drives profits to

\(^{15}\)There is strong empirical evidence on the nonlinearity of the relationship between innovative activity of a country (measured using patent data) and its R&D investment. Working with a large sample of US firm-level data, Hall et al. (1986) find an elasticity of patents to R&D of 0.5. The evidence surveyed in Kortum (1993) suggests point estimates for the patent/R&D elasticity in the range 0.1 - 0.6. More recently, Blundell et al. (2002) find a long-run elasticity of 0.5.

\(^{16}\)Eaton and Kortum (1999) adopt a similar technology in their multi-country version of the quality ladder growth model. They suggest a microfoundation for decreasing returns in innovation at the country level based on heterogeneous ability workers. As investment in innovation increases in a country, workers of lower ability will be used and productivity will decline. This microfoundation applies to this model as well.

\(^{17}\)An equally valid solution to the scale effects problem is based on the assumption that aggregate innovation becomes more difficult over time as it is spread over more varieties (see e.g. Dinopoulos and Thompson, 1998, Peretto, 1998, Howitt, 1999). Given that our focus is on inequality and not on growth, we adopt the increasing complexity specification of the difficulty index.
zero, yielding

\[ v^K(\omega, t) \left( \frac{I^K(\omega, t)}{A^K} \right)^{\frac{1-\alpha}{\alpha}} \frac{A^K}{X(\omega, t)} = w^K_H(t), \tag{22} \]

where \( v^K(\omega, t) \) is the value of a firm in sector \( \omega \) and country \( K \), and we used (20) to substitute for \( H^K(\omega, t) \). This condition states that the cost of one unit of skilled labour employed in innovation \( w^K_H(t) \) must be equal to its benefits, represented by the marginal product of labor in innovation \( A^K \) \((L^K(\omega, t)/X(\omega, t))^{1-\alpha}/X(\omega, t) \) times the prize for a successful innovation \( v^K(\omega, t) \).

Notice that, despite the leaders and followers having the same production and innovation technology, the free entry condition implies that we can compute the equilibrium focusing only on the followers’ innovation. If incumbents would innovate, they would replace themselves as market leaders, hence losing the current value of their firm. This implies that in equilibrium only followers innovate.\(^{18} \)

Efficient financial markets channel savings into innovative firms that issue a security paying the new monopoly stock market value if they win the race and zero otherwise. Since there is a continuum of industries, and simultaneous and independent innovation races, consumers can perfectly diversify away risk: the expected rate of return of a stock issued by a firm is equal to the riskless rate of return \( r(t) \). It is easy to show that this leads to the following stock market value of a firm:

\[ v^K(\omega, t) = \frac{\pi^K(\omega, t)}{r(t) + I(\omega, t) - v^K(\omega, t)/v^K(\omega, t)}, \tag{23} \]

where \( I(\omega, t) \) denotes the worldwide Poisson arrival rate of an innovation that will destroy the monopolist’s profits in industry \( \omega \). This is the Schumpeterian rate of creative destruction, which implies that the expected value of a patent is decreasing in the total innovation of the industry. Substituting for the value of the firm from (23) into (22) and using (19) to express the amount of skilled workers in terms of the innovation rate we obtain the following conditions

\[ \frac{\pi^K(\omega, t)}{r(t) + I(\omega, t) - v^K(\omega, t)/v^K(\omega, t)} \left( \frac{I^K(\omega, t)}{A^K} \right)^{\frac{1-\alpha}{\alpha}} \frac{A^K}{X(\omega, t)} = w^K_H(t), \quad \text{for } \omega \in [0, 1] \text{ and } K = D, F. \tag{24} \]

These conditions, together with the Euler equation, summarizes the utility maximizing household choice of consumption, savings, and education, and the profit maximizing choice of production and innovation. Innovation arrival rates determine the evolution of the average quality of goods in the economy \( Q_t \). Finally, in all industries, firms from both regions compete in innovation. This leads to the following decomposition of the worldwide investment in innovation, \( I(\omega, t) = I^D(\omega, t) + I^F(\omega, t) \), where \( I^D(\omega, t) \) is domestic innovation and \( I^F(\omega, t) \) is foreign innovation.

**Assumption 1 (Technology Gap).** \( A^D > A^F. \)

\(^{18}\)This is the celebrated Arrow effect, introduced by Aghion and Howitt (1992) into Schumpeterian growth theory. See Cozzi (2007) for a general interpretation.
We introduce a gap between the two countries in terms of the innovation technology parameter $A$. Since goods $\omega \in [0,1]$ are symmetric (same technologies, both in production and innovation, and enter the utility function symmetrically), the only source of structural asymmetry between the two countries is the difference between their innovation productivity. Since there is no sectorial heterogeneity in this economy, we can write, $I^D(\omega, t) = I^D(t)$ and $I^F(\omega, t) = I^F(t)$ for all $\omega$. In a steady state, the per industry probabilities of innovation per unit of time will be constant over time, which allows us to drop time indexes and write $I^D$ and $I^F$. Differentiating $Q$ with respect to time, it is straightforward to prove that:

$$ g = \frac{Q}{Q} = (\lambda^{n-1} - 1) \left( I^D + I^F \right). $$

Hence, the growth rate of aggregate quality is proportional to the global arrival rate of innovation.

### 3.4 Labour markets

The production technology specified above implies that the demand for unskilled workers is equal to the total production of goods in each national economy plus the total demand for personal services. The unskilled labour market clearing condition is

$$\left[ \Gamma(\theta^K_0) - (1 - \Gamma(\theta^K_{HS})) \right] \tilde{\theta}^K_M = \frac{(c^D + c^F)}{\lambda} q^K, $$

where $\tilde{\theta}^K_M = \int_{0}^{\theta^K_L} \theta^K d\Gamma(\theta)/ \left( \Gamma(\theta^K_0) - \Gamma(\theta^K_{LS}) \right)$ is the average ability of unskilled workers employed in production. The left-hand side is the supply of unskilled workers in manufacturing in efficiency units. The right-hand side is the demand for unskilled workers. We define $q^K = Q^K/Q$, as the share of sectors with country $K$’s leadership, where $Q^K = \int_{B^K} q(\omega)d\omega$ is the average quality of the sectors in which country $K$ has global leadership ($B^K$ is the measure of these sectors), and $q^K_D + q^K_F = 1$ by construction.$^{19}$

The market clearing condition for skilled workers is

$$\tilde{\theta}^K_H (1 - \Gamma(\theta^K_0) \phi) = \left( \frac{I^K}{A^K} \right)^{1/(1-\alpha)} \int_{0}^{1} \frac{X(\omega)}{N} d\omega = \left( \frac{I^K}{A^K} \right)^{1/(1-\alpha)} x \text{ for } K = D, F$$

where we define $x = Q^{1-\phi}/N = \int_{0}^{1} X(\omega) d\omega/N$, which is the aggregate difficulty index of innovation normalized by population. The left-hand side is the domestic supply of skilled labour (per capita) from (12), and the right-hand side is the domestic demand for skilled workers obtained after integrating equations (19) and (21).

$^{19}$At any point in time $B^K \cap B^K = [0,1]$; that is, each sector is either monopolized by a country $D$ firm or by a country $F$ firm. The measure of such sets is $q^K_D$, respectively $q^K_F$, with $q^K_D + q^K_F = 1$. Notice that each $B^K$ changes over time as firms of different countries alternate their market leadership, but in the steady state its measure is constant. Moreover, $\dot{q}^D = q^K_D I^D - (1 - q^K_D) I^K_F$, and $\dot{q}^D = 0$ if and only if $q^K_D = q^K_D I^D + q^K_F I^K_F$.
Wage Inequality. Since our main aim is to analyse the link between the technology gap and wage inequality, we need to specify the measures of inequality we want to focus on. The education choice and demand for service sector workers partition the worker/ability space as follows. Skilled workers are those whose abilities lie in \([\theta^K_{HS}, 1]\), who hire unskilled workers to perform personal services and therefore have extra time and income, and those, with ability in \([\theta^0_{0}, \theta^K_{HS}]\), who do not hire unskilled workers for performing personal services. The unskilled workers with ability in \((\theta^K_{LS}, \theta^K_{0}]\) work in the manufacturing sector, while those in \([0, \theta^K_{LS}]\) work in services. Hence, the average wage of skilled workers is the skilled wage per unit of skills \(w^D_H\) times the average skill level of educated workers:

\[
\tilde{w}^D_H = \frac{w^D_H}{1 - \Gamma(\theta^D_0)} \left[ (2 - \epsilon) \int_{\theta^D_{HS}}^{1} (\theta - \gamma_H) d\Gamma(\theta) + \int_{\theta^0_{0}}^{\theta^D_{HS}} (\theta - \gamma_H) d\Gamma(\theta) \right]. \tag{28}
\]

Notice that the skilled workers with abilities higher than \(\theta^D_{HS}\) hire service workers, thus earning the extra \(1 - \epsilon\) working time.

Similarly, the average wage of the unskilled workers employed in production is

\[
\tilde{w}^D_L = w^D_L \tilde{\theta}^D_M = w^D_L \int_{\theta^0_{LS}}^{\theta^D_{LS}} \frac{d\Gamma(\theta)}{\Gamma(\theta^D_0) - \Gamma(\theta^D_{LS})}, \tag{29}
\]

where \(\tilde{\theta}^D_M\) represents the average ability of these workers. Finally, the average wage of the service sector workers is

\[
\tilde{w}^D_S = w^D_S \tilde{\theta}^D_S = w^D_L \int_{0}^{\theta^D_{LS}} \frac{d\Gamma(\theta)}{\Gamma(\theta^D_0) - \Gamma(\theta^D_{LS})}. \tag{30}
\]

We will mainly focus on two wage gaps: the skilled/unskilled (production) workers gap and the unskilled (production)/service workers gap:

\[
\omega_H = \frac{\tilde{w}^D_H}{\tilde{w}^D_L} \text{ and } \omega_S = \frac{\tilde{w}^D_S}{\tilde{w}^D_L},
\]

both depending on the relative wage and on the relative average ability of workers.

4 Steady-State Equilibrium

A balanced growth path for this economy is an equilibrium in which per capita consumption \(c^K\), innovation \(I^K\), the share of industries with a domestic leader \(q^D\), wages \(w^D_H\) and \(w^D_L\), and the ability cutoffs \(\theta^K_{HS}\) and \(\theta^0_{0}\) are constant, while the average quality of goods \(Q\) grows at a constant rate. Since wages and sectorial innovation probabilities are constant in steady state, the free entry condition (22) and (21) imply that \(v^K(\omega)/v^K(\omega) = \dot{X}(\omega)/X(\omega) = -\phi_1 \dot{Q}/Q\), for \(K = D, F\) and for all \(\omega \in [0, 1]\). Since skill thresholds \(\theta^D_0\) and \(\theta^F_0\) are constant, from
(27) also \( Q^{1-\phi_1}/N \) has to be constant, which implies that \( \dot{Q}/Q = n/(1-\phi_1) \) and therefore \( \dot{X}(\omega)/X(\omega) = n\phi_1/(1-\phi_1) \). As a consequence, (25) implies that
\[
I^D + I^F = \frac{n}{(1-\phi_1) (\lambda^{\eta-1} - 1)}.
\]
Equation (31) dictates a long-term restriction on innovation rates based on the relative quality index and the sector sizes, thereby generalizing the "semi-endogenous growth" restrictions highlighted by Jones (1995), Kortum (1997) and Segerstrom (1998). Quite interestingly, (31) shows that our version of increasing complexity allows some degree of endogeneity in the composition of innovation rates across sectors and countries. Per capita expenditure is constant in steady state, thus the Euler equation (4) yields \( r = \rho \).

In steady state, the free entry conditions in innovation (24) take the following general form:
\[
\frac{(c^D + c^F) (\lambda - w^D_L)}{(\rho + I^D + I^F + \frac{n\phi_1}{1-\phi_1})} \lambda A^D \left( \frac{1}{x} \right) \left( \frac{I^D}{A^D} \right)^{\frac{-\alpha}{1-\alpha}} = w^D_H, \tag{32}
\]
\[
\frac{(c^D + c^F) (\lambda - 1)}{(\rho + I^D + I^F + \frac{n\phi_1}{1-\phi_1})} \lambda A^F \left( \frac{1}{x} \right) \left( \frac{I^F}{A^F} \right)^{\frac{-\alpha}{1-\alpha}} = w^F_H, \tag{33}
\]
where we have the expressions for profits specified above. Moreover, the share of industries with country \( K \) leadership is,
\[
q^K = \frac{Q^K}{Q} = \frac{I^K}{I^D + I^F}. \tag{34}
\]

To close the model we need to specify the national budget constraints. Consumption expenditure plus savings in each country equates national income, which is the value of the labour income (wages of unskilled and skilled workers) plus firm profits:
\[
c^D + w^D_H H^D = Y^D = \left[ \frac{c^D + c^F}{\lambda} w^D_L + \frac{c^D + c^F}{\lambda} (\lambda - w^D_L) \right] q^D + w^D_H H^D, \tag{35}
\]
\[
c^F + w^F_H H^F = Y^S = \left[ \frac{c^D + c^F}{\lambda} + \frac{(c^D + c^F)}{\lambda} (\lambda - 1) \right] (1 - q^D) + w^F_H H^F, \tag{36}
\]
where, since saving is equal to innovation spending, it appears both as an expenditure and as income (of the skilled workers). Moreover, to avoid double counting, we net out the wage of the personal service workers, because they are paid by their employers (the skilled workers) in order to earn additional skilled wages. Simplifying the expressions above we can write national saving as national,
\[
S^D = Y^D - c^D = c^F q^D - c^D q^F + w^D_H H^D = w^D_H H^D, \tag{37}
\]
\[
S^F = Y^F - c^F = c^D q^F - c^F q^D + w^F_H H^F = w^F_H H^F. \tag{38}
\]
\[\text{20National income simplifies into } Y^D = (c^D + c^F) q^D + w^D_H H^D, \text{ and } Y^S = (c^D + c^F) (1 - q^D) + w^F_H H^F, \text{ and using trade balance } c^F q^D = c^D q^F, \text{ we obtain the expression for saving.}\]
Moreover, equations (35) and (36) are not independent, and they lead to
\[
\frac{c^D}{c^F} = \frac{q^D}{1 - q^D},
\] (39)

The steady state equilibrium system is characterized by 13 unknowns \(c^D, c^F, I^D, I^F, w^D_L, w^H_L, \theta^D_0, \theta^F_0, \theta^D_HS, \theta^F_HS, q^D, x\), and 13 equations (6), (9), (26), (27), (31), (32), (33), (34), and (39). Below we analyse its key properties.

## 5 Analytical Results

The goal of this section is to characterize some key equilibrium properties of the model. All proofs of propositions can be found in the Appendix. We start with the following:

**Proposition 1** In the steady state equilibrium, which always exists and is unique, country \(D\) has leadership in a larger share of sectors \((q^D > q^F)\) and higher unskilled wages \((w^D_L > 1)\) than country \(F\).

The higher productivity of innovation, \(A^D > A^F\), renders the domestic country more innovative \((I^D > I^F)\), and its firms obtain market leadership in a larger range of sectors \((q^D > q^F)\), which sustains a high demand for production workers and guarantees them higher wages than in the rest of the world \((w^D_L > 1)\). The next proposition derives some comparative statics results with respect to a technological catch up by country \(F\), modelled as an increases in \(A^F\) given \(A^D\).

**Proposition 2** A reduction in the innovation technology gap \(A^D/A^F\) produces the following effects:

i. The domestic unskilled wage \(w^D_L\) decreases, the relative skilled wage \(w^D_H/w^D_L\) increases along with the fraction of the population acquiring education (lower \(\theta^D_0\)).

ii. The fraction of sectors monopolized by domestic firms \(q^D\) decreases.

iii. The fraction of the domestic labour force employed in personal services \(\theta^D_{LS}\) increases and the fraction of labour force employed in production decreases.

iv. It is harder to innovate successfully in the global economy: the innovation difficulty index, \(x\), increases.

v. Foreign skill and wage structure does not experience any change.

The effect of foreign technological competition on the skill and wage structure of the domestic country works through two different channels: the international business-stealing channel and the global competition channel. In providing the key economic intuitions, we focus on the
link between changes in the technology gap and changes in the relative wage of skilled workers. As shown in section 3.1, the relative wage \( w_D^D / w_L^D \) is the key variable that determines the whole wage and skill structure.

**International business stealing.** A reduction in the technology gap increases relative innovation intensity \( I^F / I^D \), thereby reducing the share of sectors with domestic leadership \( q^D \). This is the *business-stealing* effect, typical of Schumpeterian models, in an open economy environment with two innovating asymmetric countries. As foreign innovation technology improves, foreign firms obtain global leadership in more sectors; as a consequence, production shifts away from the home market, thereby leading to lower labour demand and lower wages for unskilled production workers. Hence, a reduction in \( w_L^D \) directly leads to higher relative skilled wage. Moreover, a reduction of unskilled wages increases domestic firm profits, thereby triggering an increase in the incentive to innovate and in the demand for skilled workers which further increases the relative skilled wage.\(^{21}\)

**Global competition effect.** Better foreign innovation technology generates an increase in the demand for skilled workers in the domestic country through its effect on the difficulty of innovation. As foreign innovation becomes relatively more productive, the relative innovation intensity \( I^F / I^D \) increases. Due to the technology gap, country \( D \) innovates more than \( F \), and since innovation technology (19) has decreasing returns at the country level, an increase in \( I^F / I^D \) raises global innovation efficiency leading to higher innovation and growth. In our semi-endogenous Schumpeterian growth model, steady-state quality growth is pinned down by population growth: combining (31) and (25) we obtain the steady-state growth rate,

\[
g = \frac{\dot{Q}}{Q} = \frac{n}{1 - \phi_1}. \tag{40}
\]

The growth-enhancing effect of a lower international technology gap can then only be temporary, and fades away in the long run due to the increase in the innovation difficulty index. Equilibrium condition (31) implies that in steady state, domestic innovation \( I^D \) decreases to exactly offset the increase in foreign innovation.\(^{22}\) The skilled labour market clearing condition (27) tells us then that, due to the increase in innovation difficulty, the domestic country is forced to devote more labour resources to innovation, and this triggers an increase in skilled wages.\(^{23}\) We name this the *global competition effect*: stronger foreign competition for innovation makes it harder for domestic firms to innovate in the global economy, thus forcing them to devote more (skilled) labour to innovation.\(^{24}\)

\(^{21}\) From (17) we can see that the domestic markup is \( (\lambda / w_L^D - 1) / \lambda \).

\(^{22}\) Recall that our numeraire is the foreign unskilled wage, hence a reduction in \( I^D \) must be interpreted as a decline in domestic innovation rate measured in terms of foreign unskilled wages.

\(^{23}\) The skilled labour market clearing condition (27) can be written as \( H(\theta_0^D) = (I^D / A^D)^{1/(1-\alpha)} x \), where \( H(\theta_0^D) \) is the supply of skilled workers. A reduction in \( A^D / A^F \) reduces \( \theta_0^D \), thereby increasing \( H(\theta_0^D) \) - since \( dH(\theta_0^D)/d\theta_0^D < 0 \). For the market to clear, the increase in the supply of skilled workers must be matched by an equivalent increase in the demand.

\(^{24}\) Modelling a world economy in which both the leading country and the followers innovate, we generate international technology leapfrogging. The existing literature on technological competition focuses on North-
Finally, because of the increase in the relative skilled wage \( w^D_H / w^D_L \), skilled workers’ time becomes more valuable, thus increasing the demand for service sector workers. The higher demand, in turns, attracts more unskilled workers into service occupations, thereby increasing the ability cutoff \( \theta^D_{LS} \). Since more people acquire education and become skilled (\( \theta^D_0 \) is lower) and more unskilled workers choose to be employed in service occupations (\( \theta^D_{LS} \) is higher), the share of unskilled workers in production shrinks.\(^{25}\)

As shown in the appendix, the equilibrium ability cutoff to acquire education for country \( F \) can be obtained in closed form as a function of parameters, \( \theta^F_0 = \theta^F_0 (n, \rho, \lambda, \eta, \phi_1, \phi, \sigma) \), and is independent on innovation efficiency parameters \( A^D \) and \( A^F \). Since \( \theta^F_0 \) is the key variable to determine the wage structure, if changes in the relative innovation efficiency do not affect this cutoff, labor market polarization remains unchanged. To interpret this result, recall that in this economy the competitive fringe pinning down the equilibrium pricing of firms is represented by the foreign country unit cost of production, the unskilled wage \( w^F_L \), which is our numeraire.

As discussed above, a reduction in the technology gap \( A^D / A^F \) reduces the unskilled wage in country \( D \), increases the profits of its firms, boosting innovation and the demand for skilled labor. In country \( F \) this mechanism does not operate: since the competitive fringe is foreign unit cost, the markup of foreign firms is \((\lambda - 1) / \lambda\), which implies that their profits are not affected by the reduction in the technology gap. As a consequence, the business-stealing effect does not lead to any change in the demand for skills, leaving the wage structure unaltered. In the extended model that we use for quantitative analysis, we show that in the presence of trade costs there exists a parameterization allowing the unit cost of both countries to potentially represent the competitive fringe. Under this parameterization the neutrality result discussed here breaks down, and changes in the technology gap affect polarization in both countries. Finally, notice that the global competition effect on the wage structure is not operative either: the increase in the difficulty index, which could potentially increase the demand for skills, is offset by the increase in the innovation efficiency \( A^F \).

\( ^{25} \)We wish to emphasize that the personal services sector in not theoretically essential for our main mechanism: in our model, foreign technological catch-up would affect domestic wage polarization even in the absence of service workers (just set \( \epsilon = 1 \)). Of course, their presence renders our stylized economy more realistic, by generating additional testable empirical predictions, allowing us to make full contact with Autor and Dorn (2013) evidence.

South models where the North innovates and the South imitates (e.g. Bonfiglioli and Gancia, 2008, Epifani and Gancia 2008, Parello, 2008, and Borota, 2012). In these models, only one of our channels, business stealing, is present but it is triggered by a different economic mechanism. Business stealing takes place through an imitation: Southern firms copy Northern technologies and then exploit their lower production costs (lower wages) to force Northern firms out of business (see e.g. Helpman, 1993). There is no scope for technological leapfrogging where firms from both countries can conquer the global market introducing higher quality products. While these models are appropriate to study North-South competition, they do not capture the nature of competition between Japanese, European and American firms in the 1970s and 1980s, which is at the core of our paper.

Having firms from both the leader and the follower country innovating is not only a necessary feature to model the type of competition we study, but it has also new theoretical implications. The global competition effect discussed above cannot be obtained in a model in which firms from the follower country can only copy frontier technologies but not innovate.
In order to keep the analysis tractable and derive the final set of properties of our model, we now assume that abilities are distributed uniformly. This assumption allows us to move from predicting the effects of changes in the technology gap on the relative wages per units of skills, to predicting the effects on average wages at the top, the middle, and the bottom of the skill distribution.

**Proposition 3** Under a uniform distribution of abilities, \( \Gamma(\theta^K_0) = \theta^K_0 \), a reduction in the innovation technology gap \( A^D/A^F \) widens wage polarization in the domestic country:

i. The wage of average skilled workers relative to that of average unskilled production workers, \( \tilde{w}_H^D/\tilde{w}_L^D \), increases.

ii. The average wage of service sector workers relative to that of unskilled production workers, \( \tilde{w}_S^D/\tilde{w}_L^D \), increases.

As shown in Proposition 2, a reduction in the innovation technology gap increases the relative wage of skilled workers \( w_H^D/w_L^F \). This effect produces changes in the average ability of skilled and unskilled workers that can potentially offset its impact on the average skilled/unskilled wage ratio \( \tilde{w}_H^D/\tilde{w}_L^D \). On the one hand, as more people acquire education (lower \( \theta^K_0 \)), the average quality of skilled workers \( \tilde{\theta}_H^D \) declines. On the other hand, a lower \( \theta^K_0 \) reduces the quality of unskilled workers employed in production while a higher \( \theta_L^{DS} \) has the opposite effect. The relative strength of these forces determines the changes in the average quality of workers \( \tilde{\theta}_H^D/\tilde{\theta}_L^D \). The result above shows that, under uniform ability distribution, the overall effect of a reduction in the technology gap on wage inequality in the upper tail of the skill distribution is positive.

Changes in the wage of service relative to production workers, \( \tilde{w}_S^D/\tilde{w}_L^D \), are pinned down by the effects on the average quality of workers in these different occupations, as the wage per unit of skills is the same for all unskilled workers. The increase in the cutoff, \( \theta_L^{DS} \), following the decline in the technology gap, increases the average quality of service workers, \( \tilde{\theta}_S^D \), but has an ambiguous effect on the average quality of production workers, as we saw above. With uniform ability distribution, the overall effect of lower technology gap on the relative service sector wage is positive. Hence, we can conclude that fiercer foreign technological competition brought about by a reduction in the innovation technology gap increases wage polarization, benefiting skilled workers and damaging workers in the middle of the skill distribution more than those at the bottom.\(^{26}\)

5.1 Globalization and the wealth to income ratio: a Schumpeterian view

Besides the predictions on the evolution of personal wage inequality, which tracks wage difference across individuals, our theory has implications for a different dimension of inequality, the

\(^{26}\)Recall that the wage per unit of skills \( w_L^D \) is the same in production and service sector occupations, and international competition has a negative effect on this wage.
wealth to income ratio. The data reported in Figure 1 show that the increase in the wealth to income ratio in the last decades is essentially due to the rise of financial assets, with housing and non-financial assets being substantially constant. More than two thirds of financial assets are represented by equities, the market value of corporations. Since the stock market value of firms is the core engine of growth in the Schumpeterian framework, our model is a good candidate to interpret the observed changes in the wealth-income ratio by linking them to the dynamics of international technological competition.

In our model, country $D$'s aggregate wealth, denoted $W^D$, coincides with the stock market value of all the profit-generating firms in the economy:

$$W^D = \tilde{v}^D q^D,$$

where

$$\tilde{v}^D \equiv \frac{(c^D + c^F) (\lambda - w^D_L)}{\left(\rho + \frac{n}{(1-\phi)}(\lambda^{\eta-1}-1) + \frac{n\phi}{1-\phi}\right) \lambda},$$

is the value of a generic patent, that is of the monopolistic firm that owns it. As in the classical Schumpeterian theory (Aghion and Howitt, 1992 and 1998), the innovation free-entry condition (24) equates the expected value of a new patent with the unit cost of innovation, here represented by the wage of a skilled labour unit$^{27}$. If we multiply both sides by the share of skilled workers in the economy $H^D$ and use (27), the left hand side becomes the aggregate value of the flow of new patents and the right side the flow of savings in the economy, as shown in (37),

$$\tilde{v}^D I^D = w^D H^D.$$ 

Since by (65) $I^D = q^D g \tilde{\lambda}$, we can write

$$\tilde{v}^D q^D g \tilde{\lambda} = w^D H^D,$$

where $\tilde{\lambda} = (\lambda^{\eta-1} - 1)^{-1}$ and $g$ given by (40) are constant. The budget constraint (37) implies that savings are $S^D = w^D_H H^D$, and expressing aggregate savings $S^D$ as the product of the ‘endogenous’ marginal propensity to save multiplied by GDP, $S^D = s^D Y^D$, transforms (42) into:

$$\tilde{v}^D \equiv \frac{W^D}{Y^D} = \frac{s^D}{g \tilde{\lambda}}.$$

This allows us to obtain the following result:

**Proposition 4** A reduction in the innovation technology gap $A^D / A^F$ raises country $D$’s wealth to national income ratio $\tilde{\beta}^D$.

$^{27}$Extending the model to the more general version - with both skilled and unskilled workers employed in innovation - used in our calibration would not change our argument.
The economic intuition is the following. Since in our semi-endogenous growth model the long-run growth rate is constant, the wealth to income ratio increases only if saving increases. As aggregate saving equals total investment in innovation, and the incentive to innovate is dictated by the market value of firms, the corporate wealth to income ratio is strictly increasing in innovation. Faster innovation in our economy has only transitional effects on growth which lead to persistently higher levels of saving and of the wealth-income ratio. Since in our model a reduction in the technology gap increases the demand for skilled workers and innovation spending in the leading country, it will also boosts the wealth to income ratio along with wage polarization.

Summary of analytical results. We have shown analytically that if the innovation productivity of the foreign country increases, reducing the gap with the world technological leader, the leading country experiences the following changes:

i. A decrease in the range of sectors enjoying global primacy.

ii. An increases in inequality at the top of the skill distribution.

iii. A decrease in inequality at the bottom of the skill distribution.

iv. An increase in personal service employment.

v. A decline in production/manufacturing employment.

vi An increase in the aggregate wealth-income ratio.

Our model seems to capture the salient stylized facts of the US labour market reported in Tables 1 and 2 and Figure 2, together with the evolution of the distribution of global patents shown in Figure 3. In the next section, we introduce additional important real world elements, which will help enrich the picture and allow us to move on to a quantitative assessment of our theory.

6 Quantitative analysis

In order to take the model to the data we generalize it along two dimensions. First, we remove the assumption that skilled workers are used only in innovation and unskilled only in production. Secondly, we assume away free trade introducing trade barriers in the form of iceberg costs. We then calibrate the parameters of the model to match some key statistics of the data discussed in section 2, compute the numerical solution using the calibrated parameters and explore the effects of our dimension of globalization on wage polarization and the wealth-income ratio.
6.1 Generalizations

We generalize the technology of our economy allowing skilled and unskilled workers to be employed in both production and innovation. The production technology is defined by the unit cost function,

\[ Z^K(w^K_L, w^K_H) = \left(1/z^K\right) \left(w^K_L\right)^\beta \left(w^K_H\right)^{1-\beta}, \text{ for } K = D, F. \]

We assume that the unit production cost in country \( F \) is the numeraire, that is \( Z^F(w^F_L, w^F_H) \equiv 1 \). The innovation technology is described by the unit cost function

\[ F^K(w^K_L, w^K_H; A^K)X(\omega) = \left(1/A^K\right) X(\omega) \left(w^K_L\right)^\varphi \left(w^K_H\right)^{1-\varphi}, \text{ for } K = D, F, \]

where the difficulty index \( X(\omega) = q(\omega)/Q^\phi \) is the same as in the benchmark model. As in the previous sections the technology gap is captured assuming \( A^D > A^F \). The country-specific production technology parameter \( z^K \) is introduced for generality, and will not play any particular role besides that of contributing to the numerical fit of the model in the calibration.

Assumption 2. (Factor Intensity): \( F^K_H / F^F_H > Z^K_H / Z^K_F \): innovation is the skill intensive activity. With the Cobb-Douglas technologies above, the factor bias of innovation is pinned down by assuming \( \beta > \varphi \).\(^{28}\)

This assumption implies less extreme factor intensity compared to the baseline model. As we will show, qualitatively it does not change the basic mechanisms: an increase in the incentive to innovate will still increase the relative demand for skilled workers, and a reduction in a country’s share of leadership will still reduce the relative demand for unskilled workers. Quantitatively, factor intensity parameters \( \beta \) and \( \varphi \) will be important to determine the size of these changes.

We also introduce trade barriers in the form of iceberg costs. Firms need to ship \( \tau > 1 \) units of goods in order to sell one unit abroad. In the presence of trade costs, the "narrow gap" assumption becomes \( \tau Z^D(w^D_L, w^D_H)/\lambda < Z^F(w^F_L, w^F_H) < Z^D(w^D_L, w^D_H) \). As in the basic model, this allows quality leaders in country \( D \) to overcome a higher production cost by supplying a higher quality good. Trade costs complicate the optimal pricing of firms compared to the basic model. The optimal price choice of country \( F \) firms selling their product domestically (that is, in country \( F \) market) \( p^F_d \), and the optimal pricing of country \( D \)'s exporting firms (that is, selling in country \( F \) market) \( p^D_x \) leads to the same limit pricing,

\[ p^F_d = \lambda Z^F(w^F_L, w^F_H) = \lambda = \lambda Z^F(w^F_L, w^F_H) = p^D_x. \]

In fact, in both cases, the limit price is anchored to the quality jump, \( \lambda \), times the unit cost of the world competitive fringe, which is country \( F \)'s production cost \( Z^F = 1 \). Slightly more complex is the optimal strategy of firms selling in country \( D \). In case \( Z^D(w^D_L, w^D_H) > \tau \), the

\(^{28}\)This assumption guarantees a strictly concave transformation curve between the production of goods and innovation probabilities. Any parameter configuration with \( \beta \neq \varphi \) would avoid a linear transformation curve. The upside is that we do not need the congestion externality parameter \( \alpha \) anymore, which can be set equal to zero at no loss.
relevant competitive fringe would still be country F firms able to enter with the previous version of the good. Then the optimal price choice of country D firms selling their product domestically (that is, in country D market) \( p_d^D \) and the price choice of country F exporting firms (that is, selling in country D market) \( p_x^F \) yield the same limit pricing,

\[
p_d^D = \tau \lambda Z^F(w_L^F, w_H^F) = \tau \lambda = \tau \lambda Z^F(w_L^F, w_H^F) = p_x^F.
\]

If instead \( Z^D(w_L^D, w_H^D) < \tau \), the relevant competitive fringe in country D market would be country D firms able to enter with the previous version of the good. Consequently, the optimal price choice of country D firms selling their product domestically and of country F exporting firms leads to the same limit pricing,

\[
p_d^D = \lambda Z^D(w_L^D, w_H^D) = p_x^F.
\]

Finally, the two pricing strategies coincide in case \( Z^D(w_L^D, w_H^D) = \tau \).

To stay close to the baseline model, in which only country F firms effectively limit-price country D firms, we add the assumption that firms in country D have to pay a small fixed cost \( \varepsilon_c > 0 \) when they are competing with a quality leader of their own country. That is, the quality leader of country D can create an additional cost (however small) to the domestic competitive fringe. For example, this could consist of small IPRs and legal barriers, distribution network and advertisement frictions, etc. We assume that this extra cost is entirely rebated to the consumers\(^ {29} \). Since firms operate under instantaneous price competition, each top-quality producer country D firm can then maximize its profits effectively being constrained only by threat of the foreign competitive fringe, not only if \( Z^D(w_L^D, w_H^D) \geq \tau \) but also if \( Z^D(w_L^D, w_H^D) < \tau \). \(^ {30} \)

Once we account for the different technologies and pricing strategies shown here, the rest of the model follows the same structure as the baseline framework. The detailed derivation can be found in the appendix. Next, we calibrate the model and explore its properties numerically.

### 6.2 Calibration

We assume that abilities are drawn from the cumulative distribution function \( \Gamma(\theta) = \theta^\varepsilon \). This is a fairly general distribution function in \([0, 1]\): when \( \varepsilon = 1 \), the ability is distributed uniformly in the population, when \( 0 < \varepsilon < 1 \) the ability distribution is skewed towards low-ability workers,

\(^ {29} \)This is just an innocuous simplification useful to eliminate \( \varepsilon_c \) from all the equations. Otherwise we would have to keep track of this overhead cost of manufacturing, with no qualitative change in the results. Notice that since \( \varepsilon_c \) can be arbitrarily small, it would not alter the calibration either, besides arbitrarily small rounding approximations.

\(^ {30} \)This assumption is not needed in the calibration, because parameters are still valid without the fixed costs. In fact, around the calibrated parameters \( Z^D(w_L^D, w_H^D) > \tau \) holds anyway - even with \( \varepsilon_c = 0 \). However, simulating the economy away enough from the calibrated parameters could at some point yield \( Z^D(w_L^D, w_H^D) < \tau \), with consequent potential modification of the competitive environment of the industries.
and for $\varepsilon > 1$ the ability distribution is log concave. In the quantitative analysis, we explore the effects of the reduction of the international technology gap on inequality in the period 1980 to 2000. The calibration period is one year. We have 17 parameters to calibrate: $\eta$, $\rho$, $n$, $\lambda$, $V$, $T_H$, $\epsilon$, $\gamma_H$, $A^D$, $A^F$, $z^D$, $z^F$, $\phi_1$, $\varphi$, $\beta$, and $\tau$.

Nine parameters $n$, $\lambda$, $V$, $T_H$, $\gamma_H$, $\phi_1$, $\tau$, $\eta$, $\rho$, have close counterparts in the economy so that their calibration is straightforward. We set $\lambda$ to 1.3, to match an average markup over the marginal cost of 30 per cent. Since the estimates of average sectorial mark-ups usually are in the interval $(0.1, 0.4)$ (Basu 1996), we take a value within this range. We choose $n$ to match a population growth rate of 1.14 percent (Bureau of labour Statistics, 1999), the total schooling time $T_H = 4$ to match the average years of college in the US, and the total working life $V = 52$ to match a life expectancy at birth for cohorts turning 18 year old in 1979 of 70 years (National Vital Report Statistics, 2010). Autor and Dorn (2013) show that the labour share of the set of occupations that we classiﬁed as skilled is lower than 40 percent in the period of interest. We follow this evidence by choosing a threshold $\gamma_H = 0.60$ to bound the share of skilled workers in our economy below 40 percent of the workforce. The current empirical literature provides a wide range of estimates for the elasticity of substitution between domestic and foreign goods. The “macro” elasticity between home and imported goods is in general smaller than the “micro” elasticity between foreign sources of imports. In a recent paper, Feenstra, Obstfeld and Russ (2012), find that the median micro elasticity is 3.1, while the macro elasticity between home and imported goods is close to one. We take a value in this range and choose $\eta = 1.2$, closer to the macro estimates. Corrado, Hulten and Sichel (2006) report an average growth of labour productivity of 1.17 percent a year in the period 1970-79. Since steady state growth is $g = n/(1 - \phi_1)$, using this value for productivity growth and the population growth rate we can back out $\phi_1 = 0.0256$. We set $\rho$, which in steady state is equal to the interest rate $r$, to 0.03 to match the risk-free rate on treasury bills. Finally, we calibrate the iceberg trade cost parameter $\tau$ using recent estimates from Novy (2013), who derive a micro-founded measure of bilateral trade costs indirectly inferring trade frictions from observable trade data. The average tariff-equivalent US bilateral trade costs with its major trading partners is found to be 74% in 1970, which corresponds to setting $\tau = 1.74$ in our

\[ 31 \text{This distribution is often used in quantitative models with heterogeneous agents, to match wage, income and earning dispersion. See e.g. Chatterjie, Corbae, Nakajima, and Rios-Rull (2007), and Antunes, Cavalcanti, Villamil (2008).} \]

\[ 32 \text{Agents choose whether to go to college at the age of 18, so that the 18 year old cohort in 1979 is represented by people born in 1961, and life expectancy at birth in 1961 in the US is 70 years. We also include retirement years into working life, by assuming that pensions are proportional (equal for simplicity) to wages during working life.} \]

\[ 33 \text{In that paper they revise US national account data to introduce investment in intangible capital, a new, more comprehensive, measure of investment in innovation. Intangible capital includes R&D, computer software, and investment in a set of activities aimed at improving existing goods, such as, advertising, design, marketing etc. The model we set up here does not have tangible (physical) capital, therefore national accounting statistics used in the calibration must be adapted to the model economy. Hence, the growth rate of productivity of 1.17\% is obtained by subtracting the share attributable to tangible capital from the overall growth rate.} \]
We normalize $A^F$ and $z^F$ to one without loss of generality. We simultaneously choose the remaining parameters, 

$$\chi \equiv [\epsilon, \varphi, \beta, \varepsilon, A^D, z^D] \in \Xi$$

so that the numerical steady-state solution of the model matches relevant statistics. The parameters are calibrated in order to minimize a loss function defined by the quadratic distance between the moments in the model and the targeted statistics. Technology parameters $\beta$, $\varphi$ and $z^D$ pin down the factor intensity of our economy and contribute to determining the relative wages $\bar{w}_{H}^D/\bar{w}_{L}^D$ and $\bar{w}_{S}^D/\bar{w}_{L}^D$. Hence we target the relative wages reported in our Table 2 from Autor and Dorn (2013). The 1980 value of ratio of the average skilled wage to average unskilled wage excluding services (1.276), the unskilled/service workers wage ratio (1.62), and the more standard skill premium, the average wage of skilled workers divided by the average wage of all unskilled workers, production plus service sector workers. The 1980 value obtained from Autor and Dorn (2013) data is 1.37. Moreover, parameters $\epsilon$, $\varepsilon$, together with $\beta$ and $\varphi$, are key in determining the occupational structure: the share of the workforce acquiring skills, the share of unskilled workers employed in production and the share of those employed in service activities. We use Autor and Dorn (2013) data targeting a share of skilled workers of 31% and a share of unskilled of 58%.

The innovation productivity parameter $A^D$ is influential in determining the geographical distribution of patents, hence we target the average 1977-80 share of US patents in the US patent office (64%) shown in the motivation section. Finally, we target the 1980 wealth-to-income ratio. The statistic consistent with our theoretical model is the Corporate Equity/National Income ratio taken from Piketty and Zucman (2014) and shown in our Figure 2.

The calibration method is the following: define $m = [m_1, ..., m_8]$ as the vector of the real data moments described above, and $y(\chi)$ the vector of equilibrium model moments generated by some vector of parameters $\chi$. Our calibrated parameter vector is obtained solving numerically

$$\hat{\chi} = \arg \min_{\chi \in \Xi} \left[ (y(\chi) - m) \cdot W \cdot (y(\chi) - m)' \right]$$

where $W$ is the weighting matrix. We use a diagonal matrix with the squares of the data targets on the diagonal and zero for all other entries; formally $W_{ii} = 1/m_i^2$. The set of possible parameter values $\Xi$ contains the minimal restrictions of non-negativity and the bound between

$^{34}$The US major trading partners included in the estimation are, Germany, Japan, UK, Canada, Korea, and Mexico. The average US bilateral trade cost is obtained using bilateral trade volumes as weights.

$^{35}$The share of service sectors workers can be obtained as a residual once these other two shares are computed.

$^{36}$Notice that the sectorial production function for innovation corresponding to cost functions $F^D$ and $F^F$ is

$$I^K = \frac{A^K \Omega}{X} (L^K)^{\varphi} (H^K)^{1-\varphi}$$

where $\Omega = \varphi^\varphi (1-\varphi)^{1-\varphi}$. The value of the total factor productivity in innovation $A^K$ is then key in determining innovation output, which in the data we can proxy with patent counts. Hence, the US patent share, represented by $I^D/I^F$ in the model is a good target for the technology gap $A^F/A^D$. 

30
0 and 1 for parameters $\epsilon$, $\varphi$, and $\beta$. The solution algorithm is as follow: for an initial vector of parameters, we solve the non-linear system of equations representing the steady-state equilibrium of our economy combining the set of algorithms in the Matlab \texttt{fsolve} function with the directed search method developed by Chris Sims and embedded in the \texttt{csolve} solver. The distance between model moments and data represented by the objective function above is minimized using a Nelder-Mead simplex algorithm. This procedure leads to the calibrated values listed in Table 3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Service Efficiency</td>
<td>$\epsilon$</td>
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</tr>
<tr>
<td>Ability Curvature</td>
<td>$\varepsilon$</td>
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</tr>
<tr>
<td>Unskill Intensity Production</td>
<td>$\beta$</td>
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</tr>
<tr>
<td>Unskill Intensity Innovation</td>
<td>$\varphi$</td>
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</tr>
<tr>
<td>Innovation Technology</td>
<td>$A^D$</td>
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</tr>
<tr>
<td>Production Technology</td>
<td>$z^D$</td>
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</tr>
</tbody>
</table>

The fit of the calibrated model is shown in Table 4,

<table>
<thead>
<tr>
<th>Target</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skill/Unskilled Wage</td>
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<td>1.40</td>
</tr>
<tr>
<td>Skill Premium</td>
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<td>1.469</td>
</tr>
<tr>
<td>Unskilled/Service Wage</td>
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<td>1.39</td>
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<td>Skilled Workers %</td>
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<td>0.30</td>
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<tr>
<td>Unskilled Workers %</td>
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<td>0.569</td>
</tr>
<tr>
<td>Service Workers %</td>
<td>0.099</td>
<td>0.097</td>
</tr>
<tr>
<td>US Patents %</td>
<td>0.63</td>
<td>0.63</td>
</tr>
<tr>
<td>Wealth to income ratio</td>
<td>0.67</td>
<td>0.67</td>
</tr>
</tbody>
</table>

where we use unskilled workers (and wages) to indicate the non-educated workers employed outside personal service occupations (production and innovation), while service workers (and wages) refer to unskilled workers employed in services. The model captures fairly well most features of the data, providing a good fit of the targets.

### 6.3 Globalization and wage polarization

Here we show the effects of globalization, in the form of increasing foreign technological competition, on the wage and occupational structure. In Figure 4, we report the effects of an increase in foreign innovation productivity $A^F$. This exercise shows how the steady state of our model reacts to reductions in the technology gap. Quantitatively, the change in the gap is chosen to reproduce the reduction in US leadership in patenting observed in the data. More precisely, we increase $A^F$ in order to match the reduction in the US share of patents from its benchmark value of 63% in 1977-80 to an average of about 53% in 1990-00. Instead of
comparing two steady states, corresponding to the initial and final value of $A^F$, we show our key endogenous variables for each intermediate value of the technology gap to provide a more complete numerical characterization of this comparative statics.

The key analytical results of the baseline model seem to be confirmed in the numerical simulation of the generalized model: the shares of skilled and service sector workers increase while the share of unskilled workers (excluding services) decreases. Inequality at the top of the ability distribution, the skilled/unskilled relative wage, increases while inequality at the bottom, the unskilled/service wage ratio, decreases. Hence, the reduction in the technology gap reproduces qualitatively the polarization of US wages and occupations documented by Autor and Dorn (2013). The economic mechanisms connecting the technology gap and the wage structure are, as in the baseline model, the business stealing and the global competition effect. The former can be observed in the reduction in the US patent share, which in the model is represented by the share of sectors with domestic leadership $q^D$. The global competition effect can be seen in the sharp increase in the innovation difficulty index $x$. Finally, notice that since in this generalized model, as in the baseline model, only foreign firms can limit-price country $D$ firms, changes in the technology gap do not affect country $F$ polarization. As discussed in Section 5, the business stealing effect cannot affect the wage and the skill structure in country $F$, as it does not have an impact on firms’ markup. The global competition effect is not operative either since the potentially higher demand for skills triggered by the increase in innovation difficulty is offset by the improvement in the relative innovation efficiency.
Table 5 shows that quantitatively the reduction of the technology gap observed in the data can reproduce about 13% of the increase in wage inequality at the upper tails of the skill distribution (the skilled-unskilled wage ratio), about 26% of the decrease in inequality at the bottom of the skill distribution (service/unskilled ratio), and 23% of the increase in the skill premium documented by Autor and Dorn (2013). These results suggest that globalization, in the form of fiercer foreign technological competition, can be an important source of wage polarization. Certainly, other channels such as routine tasks-replacing technical change and other dimensions of globalization have contributed to the recent evolution of labour market polarization. Our mechanism provides a new channel that complements existing explanations of these important facts.
Table 5. Quantitative results

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>Skill/Unskilled Wage</td>
<td>1.28</td>
<td>1.40</td>
<td>1.494</td>
<td>1.447</td>
</tr>
<tr>
<td>Skill Premium</td>
<td>1.369</td>
<td>1.469</td>
<td>1.608</td>
<td>1.528</td>
</tr>
<tr>
<td>Unskilled/Service Wage</td>
<td>1.62</td>
<td>1.39</td>
<td>1.465</td>
<td>1.338</td>
</tr>
<tr>
<td>Skilled Workers %</td>
<td>0.315</td>
<td>0.30</td>
<td>0.396</td>
<td>0.384</td>
</tr>
<tr>
<td>Unskilled Workers %</td>
<td>0.585</td>
<td>0.569</td>
<td>0.487</td>
<td>0.452</td>
</tr>
<tr>
<td>Service Workers %</td>
<td>0.099</td>
<td>0.097</td>
<td>0.115</td>
<td>0.120</td>
</tr>
<tr>
<td>Wealth to Income Ratio</td>
<td>0.67</td>
<td>0.67</td>
<td>2.82</td>
<td>2.60</td>
</tr>
</tbody>
</table>

Finally, our model allows us to replicate a sizable share of the evolution of the wealth to income ratio documented by Piketty and Zucman (2014). Our economic mechanism operates through the two key forces highlighted before. First, an increase in the rest of the world competitiveness reduces the wage of the US unskilled workers - according to the business stealing effect - and raises US firms’ profits. The rise in corporate profits boosts corporate equity, which represents wealth in our economy. Second, since stronger innovation worldwide leads to an increase in technological difficulty of innovation (our endogenous variable $x$), the expected discounted value of firms’s profits has to be higher in order to repay for the higher cost of innovation - following the global competition effect highlighted before. Hence, stronger foreign competition makes it harder for firms to successfully innovate in the global marketplace, thereby triggering more investment in innovation and fuelling an increase in the wedge between profits/wealth and labour incomes.

Although the scope of the paper is not to explain the dynamics of inequality in the foreign region, it is worth highlighting that the model’s predictions are consistent with the evidence on inequality in several advanced countries that were catching up with the US during our period of analysis. In the literature, there is substantial evidence that the skill premium and other key measures of wage inequality are mostly stable or slightly decreasing in these countries. Fuchs-Schundeln, Krueger, and Sommer (2010) find a stable education premium and a stable residual wage variance in Germany between 1982 and 1995. Similar results can be found in Pijoan-Mas and Sanchez-Marcos (2010) for Spain, but the data are limited to the 1990s. Domenej and Floden (2010) report declines of both measures of inequality for Sweden in the period 1975-95. Kambayashi et al. (2008), Kawaguchi and Mori (2008), and Lise, Sudo, Suzuki, Yamada and Yamada (2014) show that the education premium and residual wage inequality in Japan are stable or slightly declining in our period of analysis. Jappelli and Pistaferri (2010) find a stable education premium in Italy.\(^{37}\) Hence the stability of inequality in these countries is fairly well captured by our model.

\(^{37}\)The January 2010 special edition of the *Review of Economic Dynamics* titled “Cross-Sectional Facts for Macroeconomists”, contains most of these and many other country studies, and represents a rich source of empirical evidence and data for cross-country macroeconomic analysis of wage and income inequality.
For completeness, in the appendix we show that if we move away from our benchmark calibration and consider the case $Z^D(w^D_L, w^D_H) < \tau$, the model can generate a reduction in all measures of inequality in country $F$, in response to decreases in the technology gap. As explained above, when trade costs are large both countries unit costs can potentially represent the competitive fringe that pins down equilibrium limit pricing. Hence, the business-stealing effect becomes operative in country $F$ as well, but works in the opposite direction compared to country $D$. Foreign business stealing reduces production costs in country $D$, thereby increasing firm profits and the incentives to innovate. Conversely, as country $F$’s firms steal country $D$’s market shares, foreign production wages and unit costs increase, thus leading to lower profits and lower innovation incentives. It follows that the effects of a reduction in the technology gap on the demand for skills and on wage polarization is opposite for the two countries.

7 Conclusion

Technological catch-up with the United States has been a salient feature of the post-WWII global economy. Several countries have increased their ability to compete with the US and successfully entered innovation races for global market leadership: France, Germany, Japan, and South Korea earlier, and more recently China. This catching-up process is in part reflected in a particular dimension of globalization we zero in on, in which lagging countries enter global innovation races and effectively compete with technological leaders. We focus on this form of globalization not only because it has been studied less extensively than other forms (i.e. declining trade barriers), but also because technological competition, as highlighted by Schumpeterian growth theory, is of key importance in shaping the long-run distribution of market leadership in the global economy.

Our analysis shows that such technological globalization process plays a role in driving several dimensions of inequality. The decrease in competitiveness of the US innovation system determines an erosion of the US industrial base, which depresses the unskilled wages and raises the remuneration of skilled workers. More innovation in the rest of the world also raises the difficulty of innovation. As innovation becomes harder, more resources must be devoted to it in order to compete in the global playing field. Thus the demand for high skilled workers, which are used more intensively in innovation, increases together with their remuneration. As skilled labour time becomes more valuable, personal services that free labour time are more in demand, with consequent upward pressure on their wages and employment levels. Hence, globalization leads to wage polarization, to the advantage of the upper and lower tails of the skill distribution and to the detriment of occupations performed by workers with intermediate levels of skills. When matched to the US data, our model reproduces a non-negligible part of the increase in wage polarization that occurred in the US in the 1980-2000 period.

An additional prediction of our model is that more intense foreign technological competition increases the wealth-income ratio, in line with recent evidence: globalization increases
corporate profits and the market value of leading firms by reducing labour costs and increasing the competition for global market leadership. For this reason, inequality between labour and capital or wealth increases to the advantage of the latter. Since firms’ profitability pins down the value of a patent in our Schumpeterian economy, globalization triggers more innovation, faster technological progress and a higher wealth to income ratio. Our Schumpeterian framework suggests that innovation and growth, rather than economic stagnation\footnote{For an opposite view see Piketty (2014) and Piketty and Zucman (2014).}, are the ultimate sources of inequality.
8 Appendix A: proofs

Steady state leadership distribution. Considering $B^K$, the subset of sectors in which country $K$ has market leadership, the average quality of goods with domestic leadership evolves as follows

\[
\dot{Q}^D = \left( \int_{B^D} \lambda^{(j+1)(\eta-1)} d\omega + \int_{B^F} (\lambda^{(j+1)(\eta-1)} - \lambda^{(j)(\eta-1)}) d\omega \right) I^D - \int_{B^D} \lambda^{(j+1)(\eta-1)} I^F d\omega,
\]

while the average quality of industries with foreign leadership follows

\[
\dot{Q}^F = \left( \int_{B^D} \lambda^{(j+1)(\eta-1)} d\omega + \int_{B^F} (\lambda^{(j+1)(\eta-1)} - \lambda^{(j)(\eta-1)}) d\omega \right) I^F - \int_{B^F} \lambda^{(j+1)(\eta-1)} I^D d\omega,
\]

In equilibrium $\dot{Q}^D/Q^D = \dot{Q}^F/Q^F = \dot{Q}/Q$. Using the definitions $Q = Q^D + Q^F$ and $q^K = Q^K/Q$ we can write $\dot{Q}^D/Q^D = \dot{Q}^F/Q^F$ as

\[
\lambda \frac{1-q^K}{q^K} I^D + (\lambda - 1) I^D - I^F = \lambda \frac{q^K}{1-q^K} I^F + (\lambda - 1) I^F - I^D
\]

which leads to $I^D/I^F = q^K / (1 - q^K)$, and rearranging we obtain (34).

**Proof of Proposition 1.** Diving both skilled labour market clearing conditions (27) we obtain

\[
\frac{\bar{\theta}_H(\theta_0^D)}{\bar{\theta}_H(\theta_0^F)} \frac{(1 - \Gamma(\theta_0^D))}{(1 - \Gamma(\theta_0^F))} = \left( \frac{I^D/I^F}{A^D/A^F} \right)^{1/(1-\alpha)}
\]

The steady-state manufacturing labour market clearing condition for the domestic market is

\[
M(\theta_0^D) = \int_{\theta_{LS}^D} \theta d\Gamma(\theta) = \frac{c^D + c^F}{\lambda} I^D_n / (I^D_n + I^F)
\]

where we have used the definition of $\bar{\theta}_D$ and (8) to substitute out $1 - \Gamma(\theta_{HS}^D)$ and (34). Similarly for the foreign region we have

\[
M(\theta_0^F) = \int_{\theta_{LS}^F} \theta d\Gamma(\theta) = \frac{c^D + c^F}{\lambda} I^F / (I^D_n + I^F).
\]

Dividing these two equations we obtain

\[
\frac{M^D(\theta_0^D)}{M^F(\theta_0^F)} = I^D/I^F.
\]

Since from (10) we know that $\theta_{HS}^K$ is an increasing function of $\theta_0^K$, and $\theta_{LS}^K$ is an decreasing in $\theta_0^K$, we can establish that $dM(\theta_0^K)/d\theta_0^K = \theta_0^K - \theta_0^K d\theta_{LS}^K/d\theta_0^K > 0$. Hence, $M(\theta_0^K)$ increasing in $\theta_0^K$. Using (47) we can rewrite (44) as

\[
\frac{\bar{\theta}_H(\theta_0^D)}{\bar{\theta}_H(\theta_0^F)} \frac{(1 - \Gamma(\theta_0^D))}{(1 - \Gamma(\theta_0^F))} = \left( \frac{M^D(\theta_0^D)/M^F(\theta_0^D)}{A^D/A^F} \right)^{1/(1-\alpha)}
\]

37
or

\[
\frac{A^D}{A^F} = \left[ \frac{\theta_H^F(\theta_0^F) (1 - \Gamma(\theta_0^F))}{\theta_H^D(\theta_0^D) (1 - \Gamma(\theta_0^D))} \right]^{1-\alpha} \frac{M^D(\theta_0^D)}{M^F(\theta_0^F)}. \tag{48}
\]

Since \(M(\theta_0^K)\) is increasing in \(\theta_0^K\), our technology gap assumption, \(A^D > A^F\), and (48) imply that \(\theta_0^D > \theta_0^F\). Moreover (47) implies \(I^D > I^F\); hence \(q^D = I^D / (I^D_n + I^F) > 1/2\), which proves that country \(D\) has leadership in a larger share of sectors. Dividing both sides of equations (32) and (33) gives:

\[
\frac{\lambda - w^D_L}{\lambda - 1} \left( \frac{A^D}{A^F} \right)^{1-\alpha} \left( \frac{I^F}{I^D} \right)^{\alpha} = \frac{w^D_H}{w^F_H},
\]

which can be rewritten as,

\[
\frac{1}{\lambda - 1} \left( \frac{1}{w^D_L} - 1 \right) \left( \frac{A^D}{A^F} \right)^{1-\alpha} \left( \frac{I^F}{I^D} \right)^{\alpha} = \frac{(w^D_H/w^D_L)/w^F_H}{w^F_H},
\]

or as

\[
\frac{1}{\lambda - 1} \left( \frac{\lambda}{w^D_L} - 1 \right) \frac{\theta_H^F(\theta_0^F) (1 - \Gamma(\theta_0^F))}{\theta_H^D(\theta_0^D) (1 - \Gamma(\theta_0^D))} \frac{M^D(\theta_0^D)}{M^F(\theta_0^F)} = \frac{(w^D_H/w^D_L)/w^F_H}{w^F_H}, \tag{50}
\]

in which we notice the following: since \(\theta_0^D > \theta_0^F\) we must have

\[
\left[ \frac{\theta_H^F(\theta_0^F) (1 - \Gamma(\theta_0^F))}{\theta_H^D(\theta_0^D) (1 - \Gamma(\theta_0^D))} \right] \frac{M^D(\theta_0^D)}{M^F(\theta_0^F)} > 1,
\]

however, for (6), \(\theta_0^D > \theta_0^F\) also implies \((w^D_H/w^D_L)/w^F_H < 1\); in order to guarantee equality, the term \(\left( \frac{1}{w^D_L} - 1 \right)\) must be lower than 1, which only happens if \(w^D_L > 1 = w^F_L\). Hence, \(A^D > A^F\) implies \(\theta_0^D > \theta_0^F\), which leads to \((w^D_H/w^D_L)/w^F_H < 1\) and \(w^D_L > 1 = w^F_L\).

Next we prove existence and uniqueness. Notice that equations (31) and (47) now imply that

\[
I^F = \frac{n}{(1 - \phi_1) \left( \lambda^{\eta-1} - 1 \right) \left( \frac{M^D(\theta_0^D)}{M^F(\theta_0^F)} + 1 \right)} \quad \text{and} \quad I^D = \frac{n}{(1 - \phi_1) \left( \lambda^{\eta-1} - 1 \right) \left( \frac{M^F(\theta_0^F)}{M^D(\theta_0^D)} + 1 \right)}, \tag{51, 52}
\]

Hence we can write:

\[
\frac{I^D}{I^D + I^F} = \frac{M^D(\theta_0^D)}{M^D(\theta_0^D) + M^F(\theta_0^F)}
\]

and

\[
\frac{I^F}{I^D + I^F} = \frac{M^F(\theta_0^F)}{M^D(\theta_0^D) + M^F(\theta_0^F)}
\]

The domestic unskilled labour market clearing conditions (45) and (46) can be written as

\[
\lambda \left[ M^D(\theta_0^D) + M^F(\theta_0^F) \right] = c^D + c^F. \tag{53}
\]
To simplify notation, from now on we will denote aggregate per capita domestic human capital as \( H(\theta_0^D) \) and aggregate per capita foreign human capital as \( H(\theta_0^F) \). Let us now rewrite the skilled labour market conditions in the implicit form

\[
H(\theta_0^D) = \left( \frac{I^D}{A^D} \right)^{1/(1-\alpha)} x
\]

and

\[
H(\theta_0^F) = \left( \frac{I^F}{A^F} \right)^{1/(1-\alpha)} x,
\]

where \( x = Q^{1-\phi_1}/N \), and the per capita supply of skills \( H(\theta_0^D) = \tilde{\gamma}_H^K [1 - \Gamma(\theta_0^K)] \) is decreasing in the cutoff ability level \( \theta_0^D \). We can rewrite the free entry conditions in innovation equations (32) and (33) as:

\[
\begin{align*}
M^D(\theta_0^D) + M^F(\theta_0^F) & \left( \lambda - w^D_x \right) A^D x^{-1} \left( \frac{I^D}{A^D} \right)^{1-\alpha} = w^D_x \sigma_1 \left( \frac{\theta_0^D}{\theta_0^D - \gamma_H} \right), \\
M^D(\theta_0^D) + M^F(\theta_0^F) & \left( \lambda - 1 \right) A^F x^{-1} \left( \frac{I^F}{A^F} \right)^{1-\alpha} = \sigma_1 \left( \frac{\theta_0^F}{\theta_0^F - \gamma_H} \right).
\end{align*}
\]

We solve for \( x \) in equations (55):

\[
x = H(\theta_0^F) \left( \frac{I^F}{A^F} \right)^{-1/(1-\alpha)},
\]

which, plugged into free entry condition (57), and using equations (51) and simplifying, becomes:

\[
\frac{\lambda - 1}{\rho + \frac{n}{(1-\phi_1)(\lambda^{n-1}-1)} + \frac{n\phi_1}{1-\phi_1}} A^D x^{-1} \left( \frac{I^D}{A^D} \right)^{1-\alpha} = \sigma_1 \left( \frac{\theta_0^F}{\theta_0^F - \gamma_H} \right),
\]

in which the left hand side increases in \( \theta_0^F \) - due to \( M^F(\theta_0^F) \) being increasing and \( H(\theta_0^F) \) decreasing in \( \theta_0^F \) - which the right hand side decreases in \( \theta_0^F \). Hence, there is only one value of \( \theta_0^F \equiv \tilde{\theta}_0^F \) which solves this equations. Therefore equations (59) implicitly defines the unique steady state equilibrium value of \( \theta_0^F \) as a function of exogenous parameters:

\[
\theta_0^F \equiv \tilde{\theta}_0^F (n, \rho, \lambda, \eta, \phi_1, \phi, \sigma).
\]

Having proved the uniqueness of \( \theta_0^F \) facilitates our proof of the uniqueness of \( \theta_0^D \), and of all other endogenous variables, follows in a straightforward manner. In fact, the steady state equilibrium value of \( \theta_0^F \) characterized here, along with (58), allows us to rewrite the domestic human capital equilibrium condition (54) as:

\[
H(\theta_0^D) = \left( \frac{I^D}{A^D} \right)^{1/(1-\alpha)} H(\tilde{\theta}_0^F) \left( \frac{I^F}{A^F} \right)^{-1/(1-\alpha)},
\]

in which the left hand side increases in \( \theta_0^D \) - due to \( H(\tilde{\theta}_0^F) \) being increasing and \( H(\theta_0^D) \) decreasing in \( \theta_0^D \). Therefore equations (59) implicitly defines the unique steady state equilibrium value of \( \theta_0^D \) as a function of exogenous parameters:
which, after using (47) can be written as:

\[
\left( \frac{A^D}{A^F} \right)^{1/(1-\alpha)} = \left( \frac{1}{M^F(\theta^F_0)} \right)^{1/(1-\alpha)} \frac{H(\bar{\theta}^F_0)}{\sigma(\theta^D_0)},
\]

where

\[
\sigma(\theta^D_0) \equiv \left( \frac{1}{M^D(\theta^D_0)} \right)^{1/(1-\alpha)} H(\bar{\theta}^D_0).
\]

Notice that \(\sigma'(\theta^D_0) < 0\). Hence, given \(\bar{\theta}^F_0\), there is a unique value of \(\theta^D_0\) which solves equations (60). Therefore the steady state equilibrium of \(\theta^D_0\) is unique, and it is given by

\[
\theta^D_0 = \sigma^{-1}\left[ \left( \frac{1}{M^F(\bar{\theta}^F_0)} \right)^{1/(1-\alpha)} H(\bar{\theta}^F_0) \left( \frac{A^F}{A^D} \right)^{1/(1-\alpha)} \right] \equiv \theta^D_0, \left( \begin{array}{c} A^F, n, r, \lambda, \eta, \phi_1, \phi, \sigma \end{array} \right).
\]

Using the previous equation to establish uniqueness of all the other endogenous variable is straightforward.

**Proof of Proposition 2.** Equations (60) implies that

\[
\frac{\partial \theta^D_0}{\partial \left( \frac{A^F}{A^D} \right)} < 0.
\]

Since for (6) the \(w^D_H/w^D_L\) is decreasing in \(\theta^D_0\), this implies that a lower technology gap leads to higher relative skilled wage. Recalling the domestic free entry condition (56), gives

\[
\frac{[M^D(\theta^D_0) + M^F(\theta^F_0)] (\lambda - w^D_L)}{(\rho + \frac{n}{(1-\phi_1)(\lambda^{n-1}-1)} + \frac{\phi_1}{1-\phi_1})} \frac{1}{A^D x^{-1}} \left( \frac{I^D}{A^D} \right)^{\frac{1-\alpha}{1-\alpha}} = w^D_L \sigma_1 \left( \frac{\theta^D_0}{\theta^D_0 - \gamma_H} \right).
\]

using (58) to substitute for \(x\) in (62), multiplying and dividing by \(I^F\), and rearranging the right hand side, we get to

\[
\frac{[M^D(\theta^D_0) + M^F(\theta^F_0)] (\eta_1 - (\eta_1 - 1) w^D_L)}{(\rho + \frac{n}{(1-\phi_1)(\lambda^{n-1}-1)} + \frac{\phi_1}{1-\phi_1}) (\eta_1 - 1)} \left( \frac{A^D}{A^F} \right)^{\frac{1}{1-\alpha}} \frac{I^F}{H(\theta^F_0)} \left( \frac{I^F}{I^D} \right)^{\frac{\alpha}{1-\alpha}} = \left( \frac{M^F(\theta^F_0)}{M^D(\theta^D_0)} \right) \sigma_1 \left( \frac{\theta^D_0 - \gamma_H}{\theta^D_0} \right),
\]

where we have also used (51) and (47) to eliminate \(I^F\). Hence, after solving (60) for \(\left( \frac{A^D}{A^F} \right)^{1/(1-\alpha)}\), the whole free entry condition (56) can be rewritten as:

\[
\frac{n}{w^D_L} - (\eta_1 - 1) \left( \rho + \frac{n}{(1-\phi_1)(\lambda^{n-1}-1)} + \frac{\phi_1}{1-\phi_1} \right) (\eta_1 - 1) = \frac{n}{A^D} \frac{M^D(\theta^D_0)}{H(\theta^D_0)} \left( \frac{\theta^D_0 - \gamma_H}{\theta^D_0} \right) = \sigma_1,
\]

40
which, by the implicit function theorem, implies that
\[
\frac{\partial w^D_L}{\partial (\theta_0^D)} > 0,
\]
and therefore, since due to (61) \(\theta_0^D\) is decreasing in \(A^F/A^D\), we obtain
\[
\frac{\partial w^D_L}{\partial \left(\frac{A^F}{A^D}\right)} < 0. \quad (64)
\]
We have established that an increase in \(A^F/A^D\) reduces \(\theta_0^D\) and \(w^D_L\) and increases \(w^D_H/w^D_L\). This completes the proof of part (i) of the Proposition. Using result (i) and (47) we can establish
\[
\frac{\partial (I^F)}{\partial \left(\frac{A^F}{A^D}\right)} = \frac{\partial \left(\frac{\delta^K}{\delta_0^K}\right)}{\partial \left(\frac{A^F}{A^D}\right)} > 0
\]
thus proving part (ii). Furthermore, (8) and (9), allows us to conclude that a lower \(A^F/A^D\) leads to lower \(\theta^D_{HS}\) and higher \(\theta^K_{LS}\), which completes the proof of part (iii).

Finally, we need to prove that innovation difficulty, \(x\), increases. A reduction in the technology gap \(A^D/A^F\), increases the relative innovation \(I^F/I^D\) as well as the global innovation intensity \(I^F + I^D\) and the growth rate of quality but only along the transition to the steady state. The cumulated increase in aggregate quality level taking place along the transition implies that the global difficulty index \(x = Q^{1-\phi}/N\) rises until the changes in \(I^F\) and \(I^D\) exactly offset each other. Moreover, notice that dividing the steady-state innovation condition (31) by \(I^D\) we obtain
\[
\frac{1}{q^D} = \frac{n}{(1 - \phi_1)(\lambda^{\eta-1} - 1)} \frac{1}{I^D}. \quad (65)
\]
Since the fraction of sectors monopolized by its firms \(q^D\) decreases, \(I^D\) must decrease and \(I^F\) must increase by the same proportion, in order for their sum to be constant.

**Proof of Proposition 3.** The average wage of the skilled workers is
\[
\bar{w}^D_H = w^D_H \frac{1}{1 - \Gamma(\theta_0^D)} \left[ (2 - \epsilon) \int_{\theta_0^D}^{1} (\theta - \gamma_H) d\Gamma(\theta) + \int_{\theta_0^D}^{\theta_{HS}^D} (\theta - \gamma_H) d\Gamma(\theta) \right]
\]
\[
= w^D_H \frac{1}{1 - \theta_0^D} \left[ (2 - \epsilon) \left( \frac{1}{2} - \gamma_H \right) - (1 - \epsilon) \left( \frac{(\theta_{HS}^D)^2}{2} - \gamma_H \theta_{HS}^D \right) - \frac{(\theta_0^D)^2}{2} + \gamma_H \theta_0^D \right].
\]
The average wage of the unskilled workers is
\[
\bar{w}^D_L = w^D_L \int_{0}^{\theta_0^D} \frac{d\Gamma(\theta)}{\Gamma(\theta_0^D)} = \frac{w^D_L}{2} \theta_0^D.
\]
Taking the ratio we can write the skilled-unskilled (production) wage ratio as
\[
\omega_H = \frac{\bar{w}^D_H}{\bar{w}^D_L} =
\frac{2\sigma_1}{(1 - \theta_0^D) (\theta_0^D - \gamma_H)} \left[ (2 - \epsilon) \left( \frac{1}{2} - \gamma_H \right) - (1 - \epsilon) \left( \frac{(\theta_{HS}^D)^2}{2} - \gamma_H \theta_{HS}^D \right) - \frac{(\theta_0^D)^2}{2} + \gamma_H \theta_0^D \right].
\]
Notice that the numerator decreases in $\theta_0^D$ - because $\theta_0^D > \gamma_H$ and $d\theta^D_H/d\theta_0^D > 0$ - which the denominator increases in $\theta_0^D$ as long as $\theta_0^D < (1 + \gamma_H)/2$, which always holds in the US data we use. Therefore, we have $d\omega_H/d\theta_0^D < 0$. Since we have already proved that $\partial \theta_0^D / \partial (A^F/A^D) < 0$, it follows that an increase in $A^F/A^D$ increases the average relative skilled wage $\omega_H$. We can define the average wage of the unskilled workers employed in production as

$$\tilde{w}_L^D = w_L^D \int_{\theta_L^D}^{\theta_0^D} \frac{d\Gamma(\theta)}{\Gamma(\theta_0^D) - \Gamma(\theta_L^D)} = \frac{w_L^D}{2} (\theta_0^D + \theta_L^D).$$

Similarly, the average wage of service sector workers is

$$\tilde{w}_S^D = w_L^D \int_{0}^{\theta_L^S} \frac{d\Gamma(\theta)}{\Gamma(\theta_0^D) - \Gamma(\theta_L^S)} = \frac{w_L^D}{2} \theta_L^S.$$

Hence the relative service sector wage is

$$\omega_S = \frac{\tilde{w}_S^D}{\tilde{w}_L^D} = \left( \frac{1}{1 + \frac{\theta_0^D}{\theta_L^D}} \right).$$

Since an increase in $A^F/A^D$ decreases the cutoff $\theta_0^D$ and, by (8), increases $\theta_L^S$, it is easy to see that $\partial \omega_S / \partial (A^F/A^D) > 0$.

**Proof of Proposition 4.** Using (31), equation (41) becomes

$$W^D = \frac{(c^D + c^F) (\lambda - w_L^D)}{\lambda} q^D = w_H^D \left[ \frac{(I^D)^\alpha}{A^D} \right]^{1/(1-\alpha)}.$$

Using (27) and (65) national income $Y^D$ can be expressed as

$$Y^D = (c^D + c^F) q^D + \frac{nq^D}{(1 - \phi_1) (\lambda^{\nu-1} - 1)} w_H^D \left[ \frac{(I^D)^\alpha}{A^D} \right]^{1/(1-\alpha)}.$$

Dividing the last two expressions and simplifying, we obtain:

$$\frac{Y^D}{W^D} = \frac{\left( \rho + \frac{n}{(1 - \phi_1) (\lambda^{\nu-1} - 1)} + \frac{n\phi_1}{1 - \phi_1} \right) \lambda}{\lambda - w_L^D} + \frac{nq^D}{(1 - \phi_1) (\lambda^{\nu-1} - 1)},$$

which increases in $w_L^D$, which in turn is a decreasing function of $A^F$ as proved in Proposition 2.

## 9 Appendix B: derivations of the generalized model

Using the same notation as in Section 7.1, the equilibrium price choices of firms in country $D$ and in country $F$ are summarized by the following:

$$p_d^D (\omega) = p_x^F (\omega) = \min \left[ \lambda Z^D (w_L^F, w_H^F), \lambda \tau \right].$$
\[ p^F_d(\omega) = p^D_x(\omega) = \lambda Z^F(w^F_L, w^F_H) = \lambda > 1 \]

Therefore the prices paid by country D consumers depend on whether \( Z^D(w^D_L, w^D_H) > \tau \) holds. When it holds, it must be

\[ p^D_d(\omega) = \tau \lambda Z^F(w^F_L, w^F_H) = \lambda \quad \text{and} \quad p^F_x(\omega) = \tau \lambda, \]

with resulting price indexes given by

\[ P^{D(\eta-1)} = \frac{(\lambda \tau)^{\eta-1}}{Q} \quad \text{and} \quad P^{F(\eta-1)} = \frac{\lambda^{\eta-1}}{Q}. \]

If instead \( Z^D(w^D_L, w^D_H) < \tau \), then

\[ p^D_d(\omega) = \lambda Z^D(w^D_L, w^D_H) \quad \text{and} \quad p^F_x(\omega) = \lambda Z^D(w^D_L, w^D_H), \]

with resulting price indexes given by

\[ P^{D(\eta-1)} = \frac{\lambda Z^D(w^D_L, w^D_H)^{\eta-1}}{Q} \quad \text{and} \quad P^{F(\eta-1)} = \frac{\lambda^{\eta-1}}{Q}. \]

These results will affect all other steady state equilibrium conditions, as we will see more in detail now.

**Case** \( Z^D(w^D_L, w^D_H) > \tau \). In this case, the demand function for a domestic firm on the domestic market is

\[ d^D_d(\omega) = \frac{c^Dq(\omega)P^{D(\eta-1)}}{p^D_d(\omega)^\eta} = \frac{c^Dq(\omega)}{\lambda \tau Q}, \]

and its demand on the export market is

\[ d^D_x(\omega) = \frac{c^Fq(\omega)P^{F(\eta-1)}}{p^D_d(\omega)^\eta} = \frac{c^Fq(\omega)}{\lambda Q}. \]

Similarly we can derive the demand for the foreign firm, \( d^F_d(\omega) = c^Fq(\omega)/\lambda Q \) and \( d^F_x(\omega) = c^Dq(\omega)/\lambda \tau Q \). Equilibrium profits for domestic sales are

\[ \pi^D_d = \left(p^D_d - Z^D\right) d^D_d(\omega) = \left(1 - \frac{Z^D}{\lambda \tau}\right) \frac{c^Dq(\omega)N}{Q} \]

and

\[ \pi^D_x = \left(p^D_x - \tau Z^D\right) d^D_x(\omega) = \left(1 - \frac{Z^D \tau}{\lambda}\right) \frac{c^Fq(\omega)N}{Q} \]

on exports, and the \( \pi^D = \pi^D_d + \pi^D_x \) represents total profits of a domestic firms. Similarly for a foreign firm we obtain

\[ \pi^F_d = \left(1 - \frac{1}{\lambda}\right) \frac{c^Fq(\omega)N}{Q} \]

and

\[ \pi^F_x = \left(1 - \frac{\tau}{\lambda \tau}\right) \frac{c^Dq(\omega)N}{Q} = \left(1 - \frac{1}{\lambda}\right) \frac{c^Dq(\omega)N}{Q}. \]
The free entry condition in innovation races is \( v^K(\omega) = F^K(w^K_L, w^K_H) X(\omega) \), which leads to

\[
\frac{\left(1 - \frac{Z_D^D}{x} \right) c^D + \left(1 - \frac{Z_D^F}{\lambda} \right) c^F}{r + I^D + I^F + n\phi_1/(1 - \phi_1)} \frac{1}{x} = F^D(w^D_L, w^D_H)
\]

and

\[
\frac{\left(1 - \frac{1}{\lambda} \right) \left( c^F + c^D \right)}{r + I^D + I^F + n\phi_1/(1 - \phi_1)} \frac{1}{x} = F^F(w^F_L, w^F_H)
\]

where \( x = Q^{1-\phi}/N \). Labour market clearing conditions for unskilled workers are:

\[
\left[ \Gamma(\theta^D_0) - (1 - \Gamma(\theta^D_{HS})) \right] \tilde{\theta}^D_M = q^D \frac{Z^D_L}{\lambda} \left( \frac{c^D}{\tau} + \tau c^F \right) + I^D x F^D_L
\]

for the domestic country, and

\[
\left[ \Gamma(\theta^F_0) - (1 - \Gamma(\theta^F_{HS})) \right] \tilde{\theta}^F_M = (1 - q^D) \frac{Z^F_L}{\lambda} \left( c^F + c^D \right) + I^F x F^F_L.
\]

for the foreign country. Labour market clearing for skilled workers are

\[
\tilde{\theta}^D_H (1 - \Gamma(\theta^D_0)\phi) = q^D \frac{Z^D_H}{\lambda} \left( \frac{c^D}{\tau} + \tau c^F \right) + I^D x F^D_H
\]

and

\[
\tilde{\theta}^F_H (1 - \Gamma(\theta^F_0)\phi) = (1 - q^D) \frac{Z^F_H}{\lambda} \left( c^F + c^D \right) + I^F x F^F_H.
\]

The national budget constraints determine consumption in both countries,

\[
c^D = q^D \left[ \left( \frac{c^D}{\tau} + c^F \frac{Z^D_L w^D_L + Z^D_H w^D_H}{\lambda} \right) + c^D \left( 1 - \frac{Z^D}{\lambda \tau} \right) + c^F \left( 1 - \frac{Z^D}{\lambda \tau} \right) \right]
\]

and

\[
c^F = q^F \left( c^D + c^F \right) \left( \frac{Z^F_L w^F_L + Z^F_H w^F_H}{\lambda} + \left( 1 - \frac{1}{\lambda} \right) \right)
\]

where we used \( Z^D_L w^D_L = \beta Z^D \), \( Z^D_H w^D_H = (1 - \beta) Z^D \), \( Z^F_L w^F_L = \beta \), and \( Z^F_H w^F_H = 1 - \beta \) to simplify these expressions, which implode to the unique equation:

\[
\frac{c^D}{q^D} = \frac{c^F}{q^F}.
\]

The rest is the same as in the benchmark. The steady state equilibrium system is characterized by 13 unknowns, \( c^D, c^F, I^D, I^F, w^D_L, w^D_H, w^F_L, w^F_H, \theta^D_0, \theta^F_0, \theta^D_{HS}, \theta^F_{HS}, q^D, x \) and 13 equations (6), (9), (66), (67), (68), (69), (70), (71), (31), (34), and (74).
Case $Z^D(w^D_L, w^D_H) < \tau$. This case is identical to the previous case, under our assumption of a positive (however small) fixed cost for country $D$ competitive fringe firms. This is what we follow in our simulations, to constrain the generalized model to follow the model in the text more closely - that is by imposing that the relevant competitive fringe is country $F$'s. However, we can easily delineate what would happen in case the competitive fringe’s fixed cost was zero in both countries. In this case, the demand function for a domestic firm on the domestic market is

$$d^D_d(\omega) = \frac{c^D q(\omega) P^D(\eta-1)}{p^D_d(\omega)^\eta} = \frac{c^D q(\omega)}{\lambda Z^D Q}$$

and its demand on the export market is

$$d^D_x(\omega) = \frac{c^F q(\omega) P^F(\eta-1)}{p^D_d(\omega)^\eta} = \frac{c^F q(\omega)}{\lambda Q}.$$ 

Similarly we can derive the demand for the foreign firm. Equilibrium profits for domestic sales are

$$\pi^D_d = (\lambda - 1) Z^D d^D_d(\omega) = \left(1 - \frac{1}{\lambda}\right) \frac{c^D q(\omega) N}{Q}$$

and

$$\pi^D_x = (\lambda - \tau Z^D) d^D_x(\omega) = \left(1 - \frac{Z^D \tau}{\lambda}\right) \frac{c^F q(\omega) N}{Q}$$

on exports, and the $\pi^D = \pi^D_d + \pi^D_x$ represents total profits of a domestic firms. Similarly for a foreign firm we obtain

$$\pi^F_d = \left(1 - \frac{1}{\lambda}\right) \frac{c^F q(\omega) N}{Q}$$

and

$$\pi^F_x = \left(1 - \frac{\tau}{\lambda Z^D}\right) \frac{c^D q(\omega) N}{Q}.$$ 

The free entry condition in innovation races is $v^K(\omega) = F^K(w^K_L, w^K_H) X(\omega)$, which leads to

$$\frac{(1 - \frac{1}{\lambda}) c^D + \left(1 - \frac{Z^D \tau}{\lambda}\right) c^F}{r + I^D + I^F + n \phi_1 / (1 - \phi_1)} 1 = F^D(w^D_L, w^D_H)$$  

(75)

$$\frac{(1 - \frac{1}{\lambda}) c^F + \left(1 - \frac{\tau Z^D}{\lambda}\right) c^D}{r + I^D + I^F + n \phi_1 / (1 - \phi_1)} 1 = F^F(w^F_L, w^F_H)$$ 

(76)

where $x = Q^{1-\phi}/N$. Labour market clearing conditions for unskilled workers are:

$$[\Gamma(\theta^D_0) - (1 - \Gamma(\theta^D_{HS}))] \hat{\theta}^D_M = \frac{q^D Z^D_L}{\lambda} \left(\frac{c^D}{Z^D} + \tau c^F\right) + I^D x F^D_L$$ 

(77)

for the domestic country, and

$$[\Gamma(\theta^F_0) - (1 - \Gamma(\theta^F_{HS}))] \hat{\theta}^F_M = (1 - q^D) \frac{Z^F}{\lambda} \left(\frac{c^F}{Z^D} + \frac{\tau}{Z^D} c^D\right) + I^F x F^F_L.$$ 

(78)
for the foreign country. Labour market clearing for skilled workers are

\[ \tilde{\theta}_H (1 - \Gamma (\theta^D_0)) \phi = q^D \frac{Z^D_H}{\lambda} \left( \frac{c^D}{Z^D} + \tau c^F \right) + I^D x F_H^D \] (79)

and

\[ \tilde{\theta}_H (1 - \Gamma (\theta^F_0)) \phi = (1 - q^D) \frac{Z^F_H}{\lambda} \left( c^F + \frac{\tau}{Z^D} c^D \right) + I^F x F_H^F. \] (80)

The national budget constraints determine consumption in both countries,

\[ c^D = q^D \left[ \left( \frac{c^D}{Z^D} + c^F \tau \right) \frac{Z^D_L w^D_L + Z^D_H w^D_H}{\lambda} + \left( 1 - \frac{1}{\lambda} \right) c^D + \left( 1 - \frac{Z^D \tau}{\lambda} \right) c^F \right] \] (81)

\[ c^F = q^F \left[ \left( c^F + \frac{\tau}{Z^D} c^D \right) \frac{Z^F_L w^F_L + Z^F_H w^F_H}{\lambda} + \left( 1 - \frac{1}{\lambda} \right) c^F + \left( 1 - \frac{\tau}{\lambda Z^D} \right) c^D \right] \] (82)

where we used \( Z^D_L w^D_L = \beta Z^D, Z^D_H w^D_H = (1 - \beta) Z^D, Z^F_L w^F_L = \beta, \) and \( Z^F_H w^F_H = 1 - \beta \) to simplify these expressions, which simplify to the unique equation:

\[ \frac{c^D}{q^D} = \frac{c^F}{q^F}. \] (84)

The rest is the same as in the benchmark. The steady state equilibrium system is characterized by 13 unknowns, \( c^D, c^F, I^D, I^F, w^D_L, w^D_H, w^F_L, w^F_H, \theta^D_0, \theta^F_0, \theta^D_{HS}, \theta^F_{HS}, q^D, x \) and 13 equations (6), (9), (75), (76), (77), (78), (79), (80), (31), (34), and (84).

Figure A.1 shows the comparative statics with respect to \( A^F \) when we solve the model for the case \( Z^D (w^D_L, w^D_H) < \tau \). This exercise is meant to be only a sensitivity analysis to explore a further case that the model can potentially cover, so we do not re-calibrate the benchmark.
parameters.

Figure A.1. Technology Gap and Labour Market Polarization, case $Z^D(w^D_L, w^D_H) < \tau$

The opposite role of business stealing in the two countries can be understood looking at the different export markups of national firms. Country D’s firms export markup $\pi^D_x$ is positively affected by the reduction in the unit costs $Z^D$, while F firms export markup $\pi^F_x$ is negatively affected.
References


