Phasing out the GSEs

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Abstract

We develop a new model of the mortgage market where both borrowers and lenders can default. Risk tolerant savers (risk takers) act as intermediaries between risk averse depositors and impatient borrowers. The government provides mortgage guarantees and deposit insurance. Underpriced government guarantees lead to risky mortgage origination and excessive financial sector leverage. Mortgage crises frequently turn into financial crises and government bailouts due to the fragility of the intermediaries' balance sheets. Increasing the price of the mortgage guarantee crowds in the private sector, reduces financial fragility, leads to less and safer mortgage lending, lowers house prices, raises mortgage rates and risk-free interest rates. It also makes all agents in the economy strictly better off. The welfare gains are particularly large for the risk takers so that the private market solution increases wealth inequality.

JEL: G12, G15, F31.

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1 Introduction

Over the past two decades, government and quasi-government entities have dominated mortgage finance in the U.S. Fannie Mae and Freddie Mac (the GSEs), and the Federal Housing Administration have stood behind 80% of the newly originated mortgages in the past five years.[1] Ever since the collapse of the GSEs in September of 2008 and the conservatorship which socialized housing finance, there have been many proposals to bring back private capital into this market.[2] Their main idea of these policy proposals is to dramatically reduce the size and scope of the government guarantee on standard mortgages. Because this reform would turn a largely public into a largely private housing finance market, there is uncertainty and concern about its impact on house prices, the availability of mortgage finance, financial sector stability, and ultimately welfare.[3]

The question of the economic impact of wholesale mortgage finance reform requires a general equilibrium model. Such a model must recognize that residential real estate and mortgage markets have come to play a large role in the financial systems and the macroeconomy of rich countries (Jorda, Schularick, and Taylor (2014)), and that the government has a large footprint in this space. We contribute to the literature by proposing such a model where the financial sector is central.

In the benchmark model, the government enjoys a dominant position in the provision of mortgage default insurance. The model ascribes this dominance to the low insurance premium (g-fee) that the government charges banks for bearing the credit risk associated

[1]Acharya, Richardson, Van Nieuwerburgh, and White (2011) provide an in-depth discussion of the history of the GSEs, their growth, and collapse. Currently, of the $9.85 trillion stock of residential mortgages, 57% are Agency Mortgage-backed Securities guaranteed by Fannie Mae, Freddie Mac, and Ginnie Mae. Private-label mortgage backed securities make up less than 8% of the stock. The rest is unsecuritized first liens held by the GSEs and the banking sector (28%) and second liens (7%).

[2]The Obama Administration released a first report along these lines in February 2011. The bills proposed by Corker-Warner in 2013 and Johnson-Crapo in 2014 provide the most recent attempts at legislative reform.

[3]The financial and real estate industries, the Mortgage Bankers Association, and consumer advocate groups have all vehemently argued to keep a form of government guarantee in place, in the form of a public mortgage guarantor that would succeed Fannie and Freddie, for fear that there may "not be enough private capital" in the mortgage market in a fully private system, jeopardizing the stable provision of mortgage credit for a broad cross-section of households. On the other side of the spectrum, the Congressional Budget Office recently argued that housing finance reform would have minimal impact (CBO 2014).

with the underlying mortgages as well as to the minuscule amount of regulatory capital that banks must hold against guaranteed mortgage bonds. As in the real world, mortgages are long-term and non-recourse. The financial sector issues mortgages to borrowers and decides for how many to buy the government guarantee. A novel model ingredient is that the financial sector enjoys a government bailout guarantee, which is equivalent to deposit insurance. Deposit insurance is an important feature of any mortgage finance system, public or private, that the literature on mortgage finance has not considered hitherto.[4]

One of the interesting features of the model is the interaction between mortgage guarantees, which reduce the risk of banks' assets, and deposit insurance which reduces the risk of their liabilities. As relatively risk tolerant agents, bankers in the model desire a high return-high risk portfolio. By providing underpriced mortgage default insurance which the banks find attractive to buy, the riskiness of bank assets is limited. This prompts banks to increase their leverage in order to attain their desired risk-reward ratio. The favorable regulatory treatment makes guaranteed mortgage bonds excellent collateral to support this high leverage. A strong financial sector bailout guarantee further propels leverage, since it makes banks' lenders, the depositors, less sensitive to the risk of a banking collapse. A second way in which banks increase risk is through their mortgage origination decisions. They grow their mortgage portfolio and increase mortgage debt-to-income and loan-to-value ratios for borrowers. Because of the dual guarantees, banks charge low mortgage rates making borrowers happy to borrow more. In sum, the government's underpriced mortgage guarantee distorts financial sector leverage and leads to lower underwriting standards as banks pursue their desired risk-reward ratio. The financial sector bailout guarantee, or deposit insurance, amplifies these effects because it immunizes depositors from the elevated risk-taking by the bankers.

Ex-post, a riskier mortgage portfolio produces higher mortgage default rates and losses.

[4]Deposit insurance can be thought of more broadly as encompassing implicit government guarantees for short-term financial sector liabilities such as money market funds, asset-backed commercial paper, repurhcase agreements, etc. The government stepped in to rescue those markets in the Fall of 2008 and in the spring of 2009.

These losses produce deadweight costs of foreclosures. Given banks' low net worth cushion and high leverage, housing crises often turn into financial crises making banks insolvent. The economy with underpriced mortgage guarantees results in financial sector fragility as well as in higher house price volatility. However, the government absorbs the losses from housing and financial crises by issuing debt to pay for the payouts on mortgage default insurance and the financial sector bailouts. The ability to issue government debt in bad times is a technology that allows society to spread out the fiscal costs of mortgage defaults over time and to limit the negative effects on consumption smoothing. It also partially shields banks' balance sheets from the losses enabling them to continue to issue new mortgages.

A key finding is that the benefit of having the government smooth out default shocks over time is smaller than the cost of increased risk taking by the private sector. The equilibrium consumption smoothing in the economy with GSEs is poor. Intuitively, equilibrium risk sharing is poor because a fragile intermediation sector hampers the optimal flow of funds between borrowers and savers.

Capturing the spirit of the proposed reforms, our main policy experiment is to start from the "status quo" and increase the price of government mortgage insurance, thereby "crowding in" the private sector. We find that a higher guarantee fee shifts the financing of mortgages from guaranteed to private mortgage bonds. This naturally increases the riskiness of bank assets making it unnecessary for banks to go to their maximum allowed leverage ratio or to increase the riskiness of their mortgage portfolio. Intermediary net worth is higher on average so that banks have more "skin in the game" and the overall bank balance sheet is smaller. Ex-post, mortgage default and loss rates are lower in the private sector economy. Fewer housing crises turn into financial crises because of the sturdier bank balance sheets. Because of sufficient intermediary capital, banks are able to continue lending even during housing crises. The provision of mortgage credit is about equally stable in the model with and without government guarantees, dispelling the notion that the GSEs are needed to guarantee stable access to mortgage finance. The key insight is that abolition of guarantees leads banks to take less risk, moving them farther from their leverage constraints. That improves the economy's ability to allocate resources to the highest marginal utility user and thus consumption smoothing.

At g-fees that are high enough to crowd out the government completely, we find lower house prices by 5.6%, a smaller mortgage market by 12.7%, and a less levered banking sector by 9.2%. Mortgages are safer: debt-to-income ratios are 5.4% lower. The upshot in the "private market solution" is that the financial system is less fragile: the incidence of mortgage defaults and realized mortgage losses are 28.6% and 25.0% lower, and bank defaults (financial sector bailouts) are completely eliminated. The overall effect of phasing out the GSEs is that social welfare would increase the population-weighted average value function by 35%. All three groups of agents gain. Borrowers gain the least because they loose their mortgage subsidy, face higher mortgage rates, and tighter lending standards. House prices lower the wealth of existing homeowners but make it easier for new homeowners to buy a house. Risk-takers (bankers) gain the most. While abolishing the GSEs is a Pareto improvement, it increases wealth inequality.

At intermediate levels for the g-fee, we observe that the government guarantee is only taken up in bad times. This dovetails with a "mortgage insurer of last resort" option in the Obama Administration proposal which would crowd-in the government only in crises.

We plan to explore several extensions in the next draft. We will study g-fees that are contingent on the aggregate state of the economy in order to understand whether there is an interior optimum g-fee in bad times. Second, we also plan to use the model to quantitatively evaluate the 2014 Johnson-Crapo bill which proposes to put 10% private capital in front of a catastrophic government guarantee. This means that the first 10% of losses in the event of a mortgage default would be born by the private sector. The government would step in only when losses exceed that threshold. We will set the regulatory capital constraint such that banks are able to cover a 10% loss and let them choose whether or not to pay a guarantee fee to the government for insuring the residual catastrophic risk. Third, since we model mortgage bonds as long-term assets, we can explore the effect of the typical length or duration of a mortgage. Many have argued that the 30-year fixed rate mortgage, a staple of American housing finance, would not be feasible absent a government guarantee. To assess this claim, we compare the welfare effects of the guarantee for different levels of the the duration of the mortgage. The presence of guarantees allows for banks to lay off the credit risk and only hold duration risk. The government absorbs -or at least temporally transforms- mortgage credit risk. Hence, the pure term premium for long-term fixed rate mortgages may be substantially lower, with guarantees, affecting mortgage rates and house price levels. This has implications for the design of mortgage finance systems: adjustable rate or other short-duration mortgages may provide better outcomes in economies without government guarantees.

Related Literature Our paper contributes to several strands of the literature on housing, finance, and macro-economics. Unlike recent quantitative work that explores the causes and consequences of the housing boom,[5] this paper focuses on the current and future state of the housing finance system and the role the government plays in this system. It shares with these models a focus on quantitative implications and on general equilibrium considerations. In particular, house prices and interest rates are determined in equilibrium rather than exogenously specified. We simplify by working in an endowment economy with a constant housing stock.[6]

Like another strand of the literature, our model features borrowers defaulting optimally on their mortgages.[7] Unlike most of that literature, our lenders are not risk-neutral but risk averse. A default risk premium is priced into the mortgage contract which is time-varying and depends on the covariance of the risk taker's intertemporal marginal rate of substitution with the payoff on the mortgage loan. We assume that lenders impose maximum LTV ratios

[5]Examples include Ortalo-Magné and Rady (2006), Kiyotaki, Michaelides, and Nikolov (2011), Favilukis, Ludvigson, and Van Nieuwerburgh (2013), Landvoigt, Piazzesi, and Schneider (2013), Landvoigt (2012), Chu (2014). See Davis and Van Nieuwerburgh (2015) for a recent review of the literature.

[6]The role of housing supply and construction are studied in Favilukis, Ludvigson, and Van Nieuwerburgh (2013), Chatterjee and Eyigungor (2009), Hedlund (2014), and Boldrin, Garriga, Peralta-Alva, and Sanchez (2013).

[7]Recent examples of equilibrium models with default are Corbae and Quintin (2014), Garriga and Schlagenhauf (2009), Chatterjee and Eyigungor (2009), Jeske, Krueger, and Mitman (2013), Landvoigt (2012), Arslan, Guler, and Taskin (2013), and Hedlund (2014).

on borrowers, chosen to match borrower mortgage debt/income ratios in normal and housing crisis times. Unlike much of the literature, our mortgage contract is a long-term contract. This is important because of the centrality of the 30-year fixed-rate mortgage in the debate on U.S. housing finance reform. We show that our mortgage contract features the same amount of interest rate risk as the outstanding pool of mortgage-backed securities.

The biggest difference between our housing finance model and the literature is our focus on the financial sector and the role the government plays. A recent literature has emphasized the central role of financial intermediaries in the crisis.[8] Usually, intermediaries have access to a different technology from other agents. In our model, as in Drechsler, Savov, and Schnabl (2014), intermediaries arise endogenously from differences in risk aversion instead. The least risk averse savers choose to issue short-term debt ("deposits") and purchases risky long-term mortgage loans. Like in the part of the literature that emphasized debt constraints, our intermediaries face borrowing or margin constraints which link the amount of short-term liabilities they can issue to the collateral value of their assets. The net worth of the financial sector is the key state variable which governs risk sharing and asset prices in those models. In our model, intermediary wealth also is an important state variable, but it is not the only one. The wealth of the depositors, the wealth of the borrowers, and the outstanding amount of government debt all affect the equilibrium. Unlike most of the literature, we explicitly model the intermediary's decision to default. When intermediary net worth threatens to go negative, intermediaries can choose to offload all assets and liabilities onto the government. The government bailout option is equivalent to deposit insurance in the model. As emphasized above, the interaction of the mortgage guarantee and this bailout guarantee is one of the most interesting and novel aspects of our analysis. By studying the role of the financial sector in the provision of mortgages we capture the stylized fact pointed out by Jorda, Schularick, and Taylor (2014) that over a 5-year window run ups in mortgage lending and run ups in house prices raise the likelihood of a subsequent financial crises.

[8]Recent examples include Brunnermeier and Sannikov (2012), He and Krishnamurty (2013), Gârleanu and Pedersen (2011), Adrian and Boyarchenko (2012), and Maggiori (2013). Brunnermeier, Eisenbach, and Sannikov (2013) provides a review of this literature.

Mortgage and house price booms are predictive of future financial crises, and this effect has also become much more dramatic since WW2. We ask how government intervention in the form of asset guarantees or liability guarantees affects this nexus.

Finally, our paper contributes to the literature that quantifies the effect of government policies in the housing market. Most work focuses on studying the effect of abolishing the mortgage interest rate tax deductibility and the tax exemption of imputed rental income of owner-occupied housing.[9] When house and rental prices are determined endogenously, the policy changes lower house prices and price-rent ratios and cause an increase in home ownership rates. These policy changes redistribute consumption from the rich to the poor and increase welfare. Studying the GSE subsidies, Jeske, Krueger, and Mitman (2013) reach a similar conclusion regarding welfare. Our work differs from Jeske, Krueger, and Mitman (2013) in its emphasis on the role of the financial sector and the interaction between the mortgage guarantee and the bailout guarantee. We identify an effect that leads to an increase in inequality after the abolition of the mortgage subsidy, while they emphasize the regressive nature of the subsidy and hence a reduction in inequality from the abolition. In terms of model setup, there are several more differences. Jeske et al. model mortgage guarantees as a tax-financed mortgage interest rate subsidy. In our model, the government sells mortgage insurance to the private sector. Second, our mortgages are long-term in nature while theirs are one-period mortgages. Third, our lenders are risk-averse, while theirs are risk-neutral. Fourth, our model features aggregate risk but no idiosyncratic income risk, while their model has no aggregate risk and emphasizes idiosyncratic risk. Fifth, their model has a simple construction sector and constant house prices, while our model has a fixed housing supply and endogenous house prices. Sixth, their model has home ownership choice while ours does not. Seventh, our government can issue debt while theirs has to balance the budget every period. We view the robustness in conclusions as reassuring news for policymakers.

[9]For example, Gervais (2002), Chambers, Garriga, and Schlagenhauf (2009), Floetotto, Kirker, and Stroebel (2012), and Sommer and Sullivan (2013).

2 The Model

2.1 Endowments, Preferences, Technology, Timing

Endowments The model is a two-good endowment economy with a non-housing and a housing Lucas tree. The fruit of the non-housing tree, *output* Y_t, grows and its growth rate is subject to aggregate shocks. The different households are endowed with a fixed and non-tradeable share of this tree. This endowment can be interpreted as labor income. The size of the housing tree (housing stock) grows at the same stochastic trend as output. The total quantity of housing shares is fixed and normalized to 1. The housing stock yields fruit (housing services) proportional to the stock.

Preferences The model features a government and three groups of households. Impatient households are borrowers (denoted by superscript B), while patient households are savers. There are two type of savers, differentiated by their risk aversion coefficient; we refer to the less risk averse savers as "risk takers" (denoted by superscript R) and the more risk averse as "depositors" (denoted by D). Thus, for the rate of impatience we assume that $\beta_R = \beta_D > \beta_B$, and for the coefficient of relative risk aversion we assume that $\sigma_R < \sigma_B \leq \sigma_D$. All agents have Epstein-Zin preferences over the joint consumption bundle which is a Cobb-Douglas aggregate of housing and non-housing consumption with aggregation parameter θ.

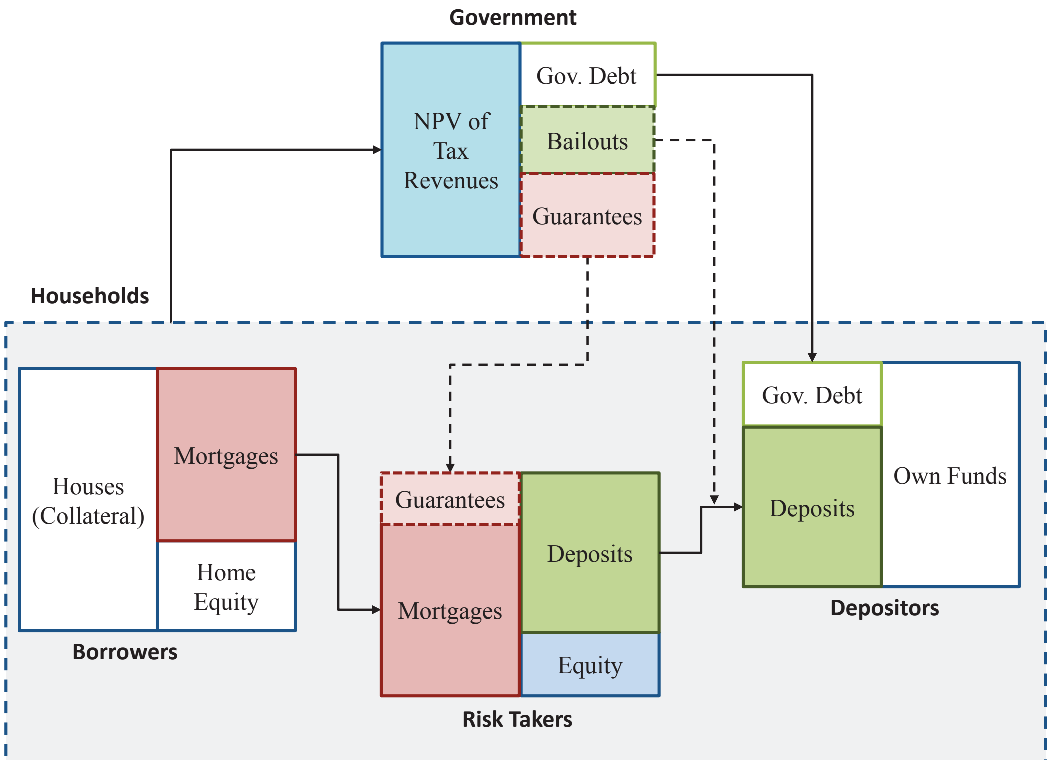
$$U_t^j = \left\{ (1-\beta_j) \left(u_t^j\right)^{1-1/\nu} + \beta_j \left(\mathrm{E}_t \left[(U_{t+1}^j)^{1-\sigma_j} \right] \right)^{\frac{1-1/\nu}{1-\sigma_j}} \right\}^{\frac{1}{1-1/\nu}} \tag{1}$$

$$u_t^j = \left(C_t^j\right)^{1-\theta} \left(A_K K_{t-1}^j\right)^{\theta} \tag{2}$$

C_j^t is numeraire non-housing consumption and the constant A_K specifies the housing services from owning the housing stock, expressed in units of the numeraiare. All agents share the same elasticity of intertemporal substitution ν.

Figure 1 depicts the balance sheets of the different agents in the economy and the flows of funds between them.

Figure 1: Balance sheets of agents in model economy



Technology In addition to the housing market and the numeraire goods market, there are three assets in the economy. The first one is a one-period short-term bond. The second one is a mortgage bond, which aggregates the mortgage loans made to all borrower households. The third one is mortgage insurance which the government sells to the private market. The guarantee turns a defaultable long-term mortgage bond into a default-free government-guaranteed mortgage bond.

Borrowers experience housing depreciation shocks and may choose to default on their mortgage. There is no recourse; savers and the government (ultimately the tax payers) bear the loss depending on whether mortgage loans are held in the form of private or government-guaranteed mortgage bonds, respectively. A novel model ingredient is that risk takers may also choose to default and declare bankruptcy. Default wipes clean their negative wealth position; the losses are absorbed by the government in a “financial sector bailout.”

Timing The timing of agents' decisions at the beginning of period t is as follows:

1. Income shocks for all types of agents and housing depreciation shocks for borrower households are realized.

2. Risk takers decide whether to declare bankruptcy. If they do, their financial wealth is set to zero and they incur a utility penalty. At the time of the decision, the magnitude of the penalty is stochastic.[10] Savers know its probability distribution and maximize expected utility by specifying a binding decision rule for each possible realization of the penalty.[11]

3. Risk takers' bankruptcy penalty is realized and they follow their decision rule from step 2. In case of bankruptcy, the government picks up the shortfall in repayments to debt holders. Borrower households decide which fraction of their mortgage debt to repay. All agents solve their consumption and portfolio choice problems and markets clear.

4. All agents consume.

Each agent's problem depends on the wealth of others; the wealth distribution is a state variable. Each agent must forecast how that state variable evolves, including the bankruptcy decisions of borrowers and risk takers. We now describe each of the three problems in detail.

2.2 Borrower's Problem

There is a representative family of borrowers, consisting of a measure one of members. Each member receives the same stochastic labor income Y_t^B, chooses the same quantity of housing k_{t-1}^B s.t. $\int_0^1 k_{t-1}^B di = K_{t-1}^B$, and the same quantity of outstanding mortgage bonds a_t^B s.t.

[10] Introducing a random utility penalty is a technical assumption we make for tractability. It makes the value function differentiable and allows us to use our numerical methods which rely on this differentiability. This randomization assumption is common in labor market models (Hansen (1985)).

[11] The assumption of making a binding default decision is necessitated in the presence of Epstein-Zin preferences.

$\int_0^1 a_t^B di = A_t^B$. The mortgage is a long-term contract, modeled as a perpetuity. Bond coupon (mortgage) payments decline geometrically, $\{1, \delta, \delta^2, \ldots\}$, where δ captures the duration of the mortgage. Labor income is subject to a tax rate, τ_t^B, and mortgage payments are tax deductible at tax rate τ_t^M.[12]

After having received income and having chosen house and mortgage size, each family member draws an idiosyncratic housing depreciation shock $\omega_{i,t} \sim F_\omega(\cdot)$ which proportionally lowers the value of the house by $(1 - \omega_{i,t}) p_t k_{t-1}^B$. The value of the house after stochastic depreciation is $\omega_{i,t} p_t k_{t-1}^B$. We denote the cross-sectional mean and standard deviation by $\mu_\omega = \mathrm{E}_i[\omega_{i,t}]$ and $\sigma_{t,\omega} = (\mathrm{Var}_i[\omega_{i,t}])^{0.5}$, where the latter can vary over time.

Each family member then optimally decides whether or not to default on the mortgage. The houses that the borrower family defaults on are turned over to (foreclosed by) the lender. Let the function $\iota(\omega) : [0, \infty) \to \{0, 1\}$ indicate the borrower's decision to default on a house of quality ω. We conjecture and later verify that the optimal default decision is characterized by a threshold level ω_t^*, such that borrowers default on all houses with $\omega_{i,t} \leq \omega_t^*$ and repay the debt for all other houses. Using the threshold level ω_t^*, we define $Z_A(\omega_t^*)$ to be the fraction of debt *repaid* to lenders and $Z_K(\omega_t^*) p_t K_{t-1}^B$ to be the value of the housing stock to the borrowers after default decisions have been made, where:

$$Z_A(\omega_t^*) = \int_0^\infty (1 - \iota(\omega)) \, f_\omega(\omega) d\omega = \Pr[\omega_{i,t} \geq \omega_t^*], \tag{3}$$

$$Z_K(\omega_t^*) = \int_0^\infty (1 - \iota(\omega)) \omega \, f_\omega(\omega) d\omega = \Pr[\omega_{i,t} \geq \omega_t^*] \; \mathrm{E}[\omega_{i,t} \,|\, \omega_{i,t} \geq \omega_t^*] \tag{4}$$

The borrower family's problem is to choose consumption C_t^B, housing K_t^B, default threshold ω_t^*, and new mortgage debt B_t^B to maximize life-time utility U_t^B in (1) subject to the

[12] In reality, mortgage interest payments are deductible at the marginal income tax rate. Because a mortgage in the model is a perpetuity, there is no clean separation of principal and interest payments. Having a different tax rate at which mortgage payments are deducted allows us to calibrate the model to a realistic level of deductibility. Appendix A discusses the mapping between our geometric mortgage bonds and mortgages in the real world in great detail.

budget constraint:

$$C_t^B + (1-\tau_t^m) Z_A(\omega_t^*) A_t^B + p_t K_t^B \leq (1-\tau_t^B) Y_t^B + Z_K(\omega_t^*) p_t K_{t-1}^B + q_t^m B_t^B, \tag{5}$$

an evolution equation for outstanding mortgage debt:

$$A_{t+1}^B = \delta Z_A\left(\omega_t^*\right) A_t^B + B_t^B, \tag{6}$$

and a maximum loan-to-value constraint:

$$\frac{A_{t+1}^B}{1-\delta} \leq \phi p_t K_t^B. \tag{7}$$

Outstanding mortgage debt at the end of the period is the sum of new borrowing B_t^B and the remaining mortgage debt at the beginning of the period after default decisions were made and mortgage payments were made on the non-defaulted (performing) part. The borrower household uses after-tax labor income, housing wealth, and new mortgage debt raised to pay for consumption, mortgage debt service (net of mortgage interest deductibility), and new home purchases. New mortgage debt raised is $q_t^m B_t^B$, where q_t^m is the price of one unit of mortgage bonds in terms of the numeraire good.

The borrowing constraint in (7) caps the face value of mortgage debt at the end of the period to a fraction of the market value of the underlying housing, where ϕ is the maximum loan-to-value ratio. With such constraint, equilibrium declines in house prices in bad times tighten borrowing constraints.

2.3 Depositors

The first type of savers, depositors, receive labor income, $Y_t^D \propto Y_t$, own a fixed share of the housing stock K_t^D, and can invest in three assets: short-term risk-free bonds, long-term private mortgage bonds, and long-term government-guaranteed mortgage bonds.

A private mortgage bond is a simple pass-through vehicle, aggregating the mortgages of the borrowers. The coupon payment on performing mortgages in the current period is $A_t^B Z_A(\omega_t^*)$. For mortgages that go in foreclosure, the saver repossesses the homes. These homes are worth $(1-\zeta)\left(\mu_\omega - Z_K(\omega_t^*)\right) p_t K_{t-1}^B$, where ζ is the fraction of home value destroyed in a foreclosure, a deadweight loss. Thus, the total payoff per unit of private mortgage bond is:

$$M_{t,P} = Z_A(\omega_t^*) + \frac{(1-\zeta)(\mu_\omega - Z_K(\omega_t^*))p_t K_{t-1}^B}{A_t^B}.$$

The price of the bond is q_t^m.

A government-guaranteed bond is a security with the same duration (maturity and cash-flow structure) as private mortgage bonds. The only difference is that it carries no mortgage default risk because of the government guarantee. To prevent having to keep track of an additional state variable, we model guarantees as one-period default insurance.[13] Combining one unit of a private mortgage bond with one unit of default insurance creates a mortgage bond that is government-guaranteed for one period. One unit of a government-guaranteed mortgage bond has the following payoff:

$$M_{t,G} = 1 + (1 - Z_A(\omega_t^*))F$$

The first term is the coupon of 1 on all loans in the pool. The second term is compensation for the loss in principal of defaulted loans. Owners of guaranteed loans receive a "principal repayment" $F = \frac{\alpha}{1-\delta}$, a constant parameter that does not depend on the value of the collateral or any state variable of the economy. We explain the choice of F below. In exchange for the guarantee, savers pay a fee $\gamma_t A_{t,G}^S$ to the government. The government sets the price of insurance γ_t.

Entering with wealth W_t^D, the depositor's problem is to choose consumption C_t^D, holdings of private mortgage bonds $A_{t+1,P}^D$, holdings of government-guaranteed mortgage bonds

[13] Rolling over default insurance every period for the life of the loan is the equivalent to the real-world guarantees provided by Fannie Mae and Freddie Mac.

$A^D_{t+1,G}$, and short-term bonds B^D_t to maximize life-time utility U^D_t in (1), subject to the budget constraint:

$$C^D_t + q^m_t A^D_{t+1,P} + (q^m_t + \gamma_t) A^D_{t+1,G} + q^f_t B^D_t + (1 - \mu_\omega) p_t K^D_{t-1} \leq (1 - \tau^S_t) Y^D_t + W^D_t \quad (8)$$

short-sales constraints on all bond holdings:

$$A^D_{t+1,P} \geq 0, \quad (9)$$

$$A^D_{t+1,G} \geq 0, \quad (10)$$

$$B^D_t \geq 0, . \quad (11)$$

The budget constraint (8) shows that the depositor uses after-tax labor income and beginning-of-period wealth to pay for consumption, purchases of private and government-guaranteed mortgage bonds and of short-term bonds, and for housing repairs. Housing repairs undo the effects of depreciation. Since the mortgage guarantee is valid for only one period, both private and government-guaranteed bonds bought last period trade for the same price q^m_t. We do not allow for negative positions in either long-term mortgage bond (equations 9 and 10). We also do not allow depositor's to to take a negative position in the short-term bond (11), consistent with our assumption that the depositor must not declare bankruptcy.

2.4 Risk Takers

After shocks to income and housing depreciation have been realized, the risk taker chooses whether or not to declare bankruptcy. Risk takers who declare bankruptcy have all their assets and liabilities liquidated. They also incur a stochastic utility penalty ρ_t, with $\rho_t \sim F_\rho$, i.i.d. over time and independent of all other shocks. At the time of the bankruptcy decision, risk takers do not yet know the realization of the bankruptcy penalty. Rather, they have to commit to a bankruptcy decision rule $D(\rho) : \mathbb{R} \rightarrow \{0, 1\}$, that specifies the optimal decision for every possible realization of ρ_t. Risk takers choose $D(\rho)$ to maximize expected utility at the beginning of the period. We conjecture and later verify that the optimal default decision is characterized by a threshold level ρ_t^*, such that risk takers default for all realizations for which the utility cost exceeds the threshold.

After the realization of the penalty, risk takers execute their bankruptcy choice according to the decision rule. They then face a consumption and portfolio choice problem identical to that of the depositor with two exceptions. First, while intertemporal preferences are still specified by equation (1), intraperiod utility u_t^j depends on the bankruptcy decision and penalty:

$$u_t^R = \frac{\left(C_t^R\right)^{1-\theta}\left(A_K K_{t-1}^R\right)^{\theta}}{\exp\left(D(\rho_t)\rho_t\right)}.$$

Second, we replace equation (11) with the following borrowing constraint:

$$-B_t^R \leq q_t^m(\xi_P A_{t+1,P}^R + \xi_G A_{t+1,G}^R). \tag{12}$$

A negative position in the short-term bond is akin to the risk taker issuing short-term bonds, or equivalently deposits. The negative position in the short-term bond must be collateralized by the market value of the risk taker's holdings of long-term mortgage bonds. The parameters ξ_P and ξ_G determine how useful private and government-guaranteed mortgage bonds are as collateral. In the calibration, we will assume that guaranteed mortgages are better collateral: $\xi_G > \xi_P$.[14]

Denote risk-taker wealth by:

$$W_t^R = (M_{t,P} + \delta Z_A(\omega_t^*) q_t^m) A_{t,P}^R + (M_{t,G} + \delta Z_A(\omega_t^*) q_t^m) A_{t,G}^R + B_{t-1}^R. \tag{13}$$

[14] In the language of finance, this short-term borrowing is exactly like a repo contract. It allows the saver to buy a mortgage bond by borrowing a fraction ξ of the purchase price while only using a fraction $1-\xi$ of the purchase price, the margin requirement, of her own capital.

2.5 Government

We model the government as set of exogenously specified tax, spending, bailout, and debt issuance policies.[15] Government tax revenues, T_t, are labor income tax receipts minus mortgage interest deduction tax expenditures plus guarantee fee receipts:

$$T_t = \tau_t^B Y_t^B + \tau_t^S (Y_t^R + Y_t^D) - \tau_t^m Z_A(\omega_t^*) A_t^B + \gamma_t (A_{t,G}^R + A_{t,G}^D)$$

Government expenditures, G_t are the sum of payouts on mortgage guarantees, financial sector bailouts, and other exogenous government spending, G_t^o:

$$G_t = (M_{t,G} - M_{t,P})(A_{t,G}^R + A_{t,G}^D) - D(\rho_t) W_t^R + G_t^o$$

The bailout to the financial sector equals the negative of the financial wealth of the risk taker, W_t^R, in the event of a bankruptcy.

The government issues one-period risk-free debt. Debt repayments and government expenditures are financed by new debt issuance and tax revenues, resulting in the budget constraint:

$$B_{t-1}^G + G_t \leq q_t^f B_t^G + T_t \tag{14}$$

We impose a transversality condition on government debt:

$$\lim_{u \to \infty} \mathrm{E}_t \left[\tilde{\mathcal{M}}_{t,t+u}^D B_{t+u}^G \right] = 0$$

where $\tilde{\mathcal{M}}^D$ is the SDF of the depositor.[16]

Because of its unique ability to tax and repay its debt, the government can spread out

[15] We consolidate the role of the GSEs and that of the Treasury department into one government, reflecting the reality as of September 2008.

[16] We show below that the risk averse saver is the marginal agent for short-term risk-free debt. In the numerical work below, we keep the ratio of government debt to GDP contained between $\underline{b^G}$ and $\overline{b^G}$ by increasing other government expenditure exponentially when debt-to-GDP threatens to fall below $\underline{b^G}$ and lowering G_t^o exponentially (until it hits zero) when debt-to-GDP threatens to exceed $\overline{b^G}$.

the cost of mortgage default waves and financial sector rescue operations over time.

Government policy parameters are $\Theta_t = \left(\tau_t^B, \tau_t^S, \tau_t^m, \gamma_t, G_t^o, \phi, \xi_G, \xi_P\right)$. The parameters ϕ in equation (7) and (ξ_G, ξ_P) in equation (12) can be thought of as macro-prudential policy tools. One could add the parameters that govern the utility cost of bankruptcy of risk takers to the set of policy levers, since the government may have some ability to control the fortunes of the financial sector in the event of a bankruptcy.

2.6 Equilibrium

Given a sequence of income shocks $\{Y_t\}$, housing depreciation shocks $\{\omega_{t,i}\}_{i\in B}$, and utility costs of default shocks ρ_t, and given a government policy Θ_t, a competitive equilibrium is an allocation $\{C_t^B, K_t^B, B_t^B\}$ for borrowers and $\{C_t^S, A_{t,P}^S, A_{t,G}^S, B_t^S\}$ for savers $S \in \{R, D\}$, default policies $\iota(\omega_{it})$ and $D(\rho_t)$, and a price vector $\{p_t, q_t^m, q_t^f\}$, such that given the prices, borrowers, depositors, and risk-takers maximize life-time utility subject to their constraints, the government satisfies its budget constraint, and markets clear.

The market clearing conditions are:

1. Risk-free bonds: $B_t^G = B_t^D + B_t^R$
2. Mortgages: $A_t^B = A_{t,G}^R + A_{t,P}^R + A_{t,G}^D + A_{t,P}^D$
3. Housing tree shares: $K_t^B + K_t^R + K_t^D = 1$
4. Consumption: $Y_t^B + Y_t^R + Y_t^D = C_t^B + C_t^R + C_t^D + (1-\mu_{t,\omega})p_t + G_t^o + \zeta(\mu_{t,\omega} - Z_K(\omega_t^*))\frac{p_t K_{t-1}^B}{A_t^B}$

The last equation states that total non-housing resources equal the sum of non-housing consumption expenditures and home renovations by the households, (wasteful) spending by the government, and lost resources due to the deadweight costs of foreclosure.

2.7 Welfare

In order to compare economies that differ in the policy parameter vector Θ_t, we must take a stance on how to weigh the different agents. We propose a utilitarian social welfare function summing value functions of the agents according to their population weights ℓ:

$$\mathcal{W}_t(\cdot;\Theta_t) = \ell^B V_t^B + \ell^D V_t^D + \ell^R V_t^R,$$

where the $V^i(\cdot)$ functions are the value functions defined in the appendix.

3 Model Solution and Calibration

3.1 First Order Conditions

Appendix A presents the Bellman equations for each of the three household types and derives first-order conditions for optimality. We highlight some key features of the solution here. First, since borrowers are the only households freely choosing their housing position, their choice pins down the price of housing in the economy. Let $\tilde{\mathcal{M}}^i_{t,t+1}$ be the intertemporal marginal rate of substitution (or stochastic discount factor) for agent $i \in \{B, D, R\}$, with expressions provided in the Appendix. At the optimum, house prices satisfy the recursion:

$$p_t \left[1 - \tilde{\lambda}_t^B \phi_t\right] = \mathrm{E}_t \left[\tilde{\mathcal{M}}^B_{t,t+1} e^{g_{t+1}} \left\{ p_{t+1} Z_K(\omega^*_{t+1}) + \frac{\theta C^B_{t+1}}{(1-\theta)K^B_t} \right\}\right] \tag{15}$$

The marginal cost of housing on the left-hand side consists of the house price p_t minus a term which reflects the collateral benefit of housing; an extra unit of housing relaxes the maximum LTV constraint (7). The right hand side captures the expected discounted future marginal benefits which depends on the resale value of the non-defaulted stock and the dividend from housing, which is the intratemporal marginal rate of substitution between housing and non-housing goods.

Second, we analyze the borrower's optimal foreclosure decision. In the appendix, we show that the optimal default threshold is given by:

$$\omega_t^* = \frac{(1-\tau_t^m + \delta q_t^m)A_t^B}{p_t K_{t-1}^B}.$$

At the threshold level ω_t^*, the cost from foreclosure, which is the loss of a house valued at $\omega_t^* p_t K_{t-1}^B$ but being released from the mortgage, exactly equals the expected cost from continuing the service the mortgage (including the option to default in the future which is encoded in q_t^m), $(1-\tau_t^m + \delta q_t^m)A_t^B$, but keeping the house. The cutoff has an intuitive interpretation. It is the aggregate loan-to-value ratio of the borrowers, with both mortgage debt and housing valued at market prices. When the market leverage of the borrower increases, default becomes more likely.

In this version of the paper, we restrict the depositor to only invest in deposits, i.e., short-term debt issued by the risk taker. This debt is equivalent to government debt by virtue of the deposit insurance. We plan to relax this assumption on the menu of investible assets for the depositor in later versions of the paper. The depositor's first-order condition for the the short-term bond is:

$$q_t^f \quad = \quad \mathrm{E}_t \left[\tilde{\mathcal{M}}_{t,t+1}^D \right]$$

The risk taker can invest in both government guaranteed and private MBS. The respective first-order conditions are:

$$\begin{aligned} q_t^m(1-\xi_G \tilde{\lambda}_t^R) + \gamma_t &= \mathrm{E}_t \left[\tilde{\mathcal{M}}_{t,t+1}^R \left(M_{G,t+1} + \delta Z_A(\omega_{t+1}^*) q_{t+1}^m \right) \right] \\ q_t^m(1-\xi_P \tilde{\lambda}_t^R) &= \mathrm{E}_t \left[\tilde{\mathcal{M}}_{t,t+1}^R \left(M_{P,t+1} + \delta Z_A(\omega_{t+1}^*) q_{t+1}^m \right) \right]. \end{aligned}$$

The marginal cost of a guaranteed mortgage bond is the price q_t^m plus the guarantee fee γ_t (expressed as a price) while the benefit is the expected discounted value of the bond tomorrow, which consists of the coupon payment and the repayment of principal in case of default (both are in M_G) plus the resale value of the non-defaulted portion of the mortgage bond. The cost is lowered by the relaxation of the margin constraints, and depends on the haircut ξ_G for guaranteed mortgages. The first-order condition for private mortgages is similar, without the guarantee fee term, with a different collateral requirement term (ξ_P), and a different mortgage payoff M_P. One way of restating the risk taker's choice is in terms of how many units of the mortgage to lend to borrowers, and for how much of these holdings to buy default insurance from the government. The optimal amount of default insurance to buy solves:

$$\gamma_t = \mathrm{E}_t \left[\tilde{\mathcal{M}}^R_{t,t+1} \left(M_{G,t+1} - M_{P,t+1} \right) \right] + \lambda^R_t q^m_t \left(\xi_G - \xi_P \right)$$

Risk takers will buy insurance until the marginal cost of insurance on the left equals the marginal benefit. An extra unit of default insurance increases the payoff of the mortgage and it increases the collateralizability of a mortgage, a benefit which only matters when the borrowing constraint binds.

The risk taker will optimally default whenever the utility costs of doing so is sufficiently small: $\rho_t < \rho^*_t$. The threshold depends on her wealth W^R_t and the state variables $\mathcal{S}^R_t$ that are exogenous to the risk taker, including the wealth of the borrower and of the depositor, and the outstanding amount of government debt. At the threshold, she is indifferent between defaulting and offloading her (negative) wealth onto the government or carrying on:

$$V^R(0, \rho^*_t, \mathcal{S}^R_t) = V^R(W^S_t, 0, \mathcal{S}^R_t).$$

In equilibrium, the risk-taker's short-sales constraint for government-guaranteed bonds will typically be binding so that her position in these bonds is zero. In contrast, she is the marginal agent for private mortgage bonds:

$$q^m_t (1 - \xi_P \tilde{\lambda}^R_t) = \mathrm{E}_t \left[\tilde{\mathcal{M}}^R_{t,t+1} \left(M_{P,t+1} + \delta Z_A(\omega^*_{t+1}) q^m_{t+1} \right) \right].$$

She finances these bonds with her own wealth but also by issuing short-term debt (deposits). The bailout guarantee that the government provides to the risk takers makes these deposits risk-free and equivalent to short-term government debt. That is, the bailout guarantee is deposit insurance. The risk-taker may or may not borrow up to the limit in (12). If she does, the Lagrange multiplier on that constraint $\tilde{\lambda}_t^R > 0$. Since mortgage bonds enable the issuance of more short-term debt, they increase risk taker demand for mortgage bonds when the constraint binds (lower the marginal cost by $q_t^m \xi_P \tilde{\lambda}_t^R$).

3.2 Calibration

The parameters of the model and their targets are summarized in Table 1.

Aggregate Income The model is calibrated at annual frequency. Aggregate endowment or labor income Y_t follows:

$$
\begin{aligned}
Y_t &= Y_{t-1}\exp(g_t) \\
g_t &= \rho_g g_{t-1} + (1-\rho_g)\bar{g} + \epsilon_t, \qquad \epsilon_t \sim iid\, \mathcal{N}(0, \sigma_g)
\end{aligned}
$$

We scale all variables by permanent income in order render the problem stationary. Given the persistence of income growth, g_t, becomes a state variable. We discretize the g_t process into a 5-state Markov chain using the method of Rouwenhorst. The procedure matches the mean, volatility, and persistence of GDP growth by choosing both the grid points and the transition probabilities between them. We use annual data on real per capita GDP growth from the BEA NIPA tables from 1929-2014 and exclude the war years 1940-1945. The resulting mean is 1.9%, the standard deviation is 3.9%, and the persistence is 0.42. The states, the transition probability matrix, and the stationary distribution are listed in Appendix B.1.

Table 1: Calibration

This table reports the parameter values of our model.

	Parameter	Description	Value	Target
		Exogenous Shocks		
1	$\bar{g}$	mean income growth	1.9%	Mean rpc GDP gr 1929-2013
2	σ_g	volatility income growth	3.9%	Vol rpc GDP gr 1929-2013
3	ρ_g	persistence income growth	0.41	AC(1) rpc GDP gr 1929-2013
4	μ_ω	mean idiosync. house value shock	2.5%	Housing depreciation Census
5	σ_ω	volatility idiosync. house value shock	{0.25,0.32}	Mortgage loss rates (Appendix B.2)
6	p^ω_{LL}	transition prob	0.2	frequency of mortgage crises of 10%
7	p^ω_{HH}	transition prob	0.99	duration of mortgage crises of 2y
		Population, Income, and Housing Shares		
8	ℓ^i, $i \in \{B, D, R\}$	population shares	{47,51,2}%	Population shares SCF 1995-2013
9	Y^i, $i \in \{B, D, R\}$	income shares	{38,52,10}%	Income shares SCF 1995-2013
10	K^i, $i \in \{B, D, R\}$	housing shares	{39,49,12}%	Housing wealth shares SCF 1995-2013
		Mortgages		
11	δ	average life of mortgage pool	0.96	Duration Fcn. (Appendix B.3)
12	α	guarantee payout fraction	0.35	Duration Fcn. (Appendix B.3)
		Preferences		
13	σ^B	risk aversion borrower	8	Vol household mortgage debt to GDP 1985-2014
14	β^B	time discount factor borrower	0.88	Mean housing wealth to GDP 1985-2014
15	θ^B	housing expenditure share	0.20	Housing expenditure share NIPA
16	σ^D	risk aversion depositor	8	volatility risk-free interest rate 1985-2014
17	$\beta^D = \beta^R$	time discount factor savers	0.975	Mean risk-free interest rate 1985-2014
18	σ^R	risk aversion risk taker	4	Financial sector leverage Flow of Funds 1985-2014
19	ν	intertemp. elasticity of subst.	1	
		Government Policy		
20	$\tau^S = \tau^B$	income tax rate	18.5%	BEA govmt revenues to trend GDP 1929-2013
21	G^o	exogenous govmt spending	16.1%	BEA govmt spending to trend GDP 1929-2013
22	τ^m	mortgage interest rate deductibility	0.65 τ^B	See text
23	ϕ	collateral constr	80%	Mean borrowers' mortgage debt-to-income SCF 1995-2013
24	ξ_G	margin guaranteed MBS	1.6%	Basel 2/3 regulatory capital charge agency MBS
25	ξ_P	margin private MBS	8%	Basel 2/3 regulatory capital charge non-agency mortgages

Foreclosure crises The stochastic depreciation shocks or idiosyncratic house value shocks, $\omega_{i,t}$, are drawn from a Gamma distribution characterized by shape and a scale parameters $(\chi_{t,0}, \chi_{t,1})$. $F_\omega(\cdot; \chi_{t,0}, \chi_{t,1})$ is the corresponding CDF. We choose $\{\chi_{t,0}, \chi_{t,1}\}$ to keep the mean μ_ω constant at 0.975, implying annual depreciation of housing of 2.5%, and to let the cross-sectional standard deviation $\sigma_{t,\omega}$ take on one of two values, a high and a low value. Together with the maximum loan-to-value parameter ϕ, $(\sigma_{H,\omega}, \sigma_{L,\omega})$ determine the expected losses from mortgage default. We set these three parameters to fit three targets: mortgage loss rate during normal times, mortgage loss rate during foreclosure crises, and average mortgage debt-to-income ratios for borrower. These targets are 0.3%, 3.4%, and 134%, respectively. Appendix B.2 explains the rationale behind these targets and the data sources. The resulting parameter values are $\phi = 0.80$, $\sigma_{H,\omega} = 0.42$, and $\sigma_{L,\omega} = 0.25$. The current calibration overstates the loss rates.

To pin down the transition probabilities of the 2-state Markov chain for $\sigma_{t,\omega}$, we assume that when the aggregate income growth rate in the current period is high (g is in one of the top three income states), there is a zero chance of transitioning from the $\sigma_{L,\omega}$ to the $\sigma_{H,\omega}$ state and a 100% chance of transitioning from the $\sigma_{H,\omega}$ to the $\sigma_{L,\omega}$ state. Conditional on low growth (g is in one of the bottom two income states) we calibrate the two transition probability parameters (rows have to sum to 1), p^ω_{LL} and p^ω_{HH}, to match the frequency and length of mortgage crises. Based on the argument by Jorda et al. (2014) that most financial crises are related to the mortgage market and based on the historical frequency of financial crises in Reinhart and Rogoff, we target a 10% probability of a foreclosure crisis. Conditional on a crisis, we set the expected length to 2 years, based on evidence in Jorda et al. and Reinhart and Rogoff. As such, the model implies that not all recessions are mortgage crises, but all mortgage crises are recessions. In a long simulation, 33% of recessions are also crises. This compares to a fraction of 6/22 (≈27%) in Jorda et al.. The correlation between $\sigma_{t,\omega}$ and g_t is -0.46. The model generates persistence in the mortgage default rate of .41, driven by the persistence of $\sigma_{t,\omega}$ of .46 (in the same simulation).

Population and wealth shares To pin down the labor income and housing shares for borrowers, depositors, and risk takers, we calculate a net fixed-income position for each household in the Survey of Consumer Finance (SCF).[17] Net fixed income equals total bond and bond-equivalent holdings minus total debt. If this position is positive, we consider a household to be a saver, otherwise it is a borrower. For savers, we calculate the amount of risky assets, defined as their holdings of stocks, business wealth, and real estate wealth, as well as the share of these risky assets in total wealth. We define risk takes as households that are within the top 5% of risky asset holdings and have a risky asset share of at least 75%. This delivers population shares of $\ell^B = 47\%$, $\ell^D = 51\%$, and $\ell^R = 2\%$. Based on this classification and the same SCF data, borrowers receive 38% of aggregate income and own 39% of residential real estate. Depositors receive 52% of income and 49% of housing wealth. Finally, risk takers receive 10% of income and 12% of housing wealth. By virtue of the calibration, the model thus matches basic aspects of the observed income and wealth inequality.

Mortgages In our model, a government-guaranteed MBS is a geometric bond. The issuer of one bond at time t promises to pay the holder 1 at time $t + 1$, δ at time $t + 2$, δ^2 at time $t+3$, and so on. If the borrower defaults on the mortgage, the government guarantee entitles the holder to receive a "principal repayment" $F = \frac{\alpha}{1-\delta}$, a constant parameter that does not depend on the value of the collateral or any state variable of the economy. We estimate values for δ and F such that the duration of the geometric mortgage matches the duration of the portfolio of outstanding mortgage-backed securities, as measured by the Barclays MBS Index, across a history of observed mortgage rates. This novel procedure recognizes that the mortgage in the model represents the pool of all outstanding mortgages of all vintages. The repayment F reflects the fact that prepayments happen when rates are low and reinvestment opportunities are poor. Appendix B.3 provides the details. We find that values of $\delta = 0.967$ and $F = 10.678$ imply a relationship between price and mortgage rate for the geometric

[17] We use all survey waves from 1995 until 2013 and average across them.

mortgage that closely matches the price-rate relationship for a real-life MBS pool consisting of fixed-rate mortgages issues across a range of vintages. The average duration in model and data of the mortgage (pool) is about 4 years. Like the real-life MBS pool, the geometric mortgage price is convex in rates when rates are high (the prepayment option is out-of-the-money) and concave when rates are low ("negative convexity" when the prepayment option is in-the-money). Thus, the geometric mortgage has the same interest rate risk (duration) of real-life mortgages for different interest rate scenarios. Despite its simplicity, the perpetual mortgage captures the key features of real-life guaranteed MBS pools.

Government parameters Government policy consists of mortgage guarantee policy and general taxation and spending policy. In our desire to have a quantitatively meaningful model, we believe it is important to also capture the latter. After the conservatorship of Fannie Mae and Freddie Mac in September 2008, the merger of the GSEs and the Treasury Department became a reality.

Starting with the guarantee policy, our parameter γ specifies the cost of a guarantee, expressed in the same units as the price of the mortgage. Real-world guarantee fees are expressed as a surcharge to the interest rate. We will consider several values for γ with implied g-fees ranging from 0 basis points to 70 basis point. Freddie Mac's management and g-fee rate has been stable at around 20bps from 2000 to 2012 and has increased gradually from 20bps at the start of 2012 to 32 bps at the end of 2014. Fannie Mae's single-family effective g-fee was also right around 20bps between 2000 and 2009, and has been increasing gradually from 20bps at the start of 2009 to 41 bps at the end of 2014 (Urban Institute Housing Finance Policy Center, December 2014 update).

We set the proportional income tax rate equal to $\tau^S = \tau^B = 0.185$ in order to match average tax revenue to trend GDP in the 1929-2013 U.S. data of 18.8%. We set exogenous government spending equal to $G^o = 0.161$ (times trend GDP of 1) in order to match average exogenous government spending to trend GDP in the 1929-2013 U.S. data of 16.1%.[18] As a

[18]The data are from Table 3.1 from the BEA. Exogenous government spending is defined as consumption

fraction of realized GDP, expenditures fluctuate, mimicking their counter-cyclicality in the data. Tax revenues are pro-cyclical, as in the data. Every dollar of income is taxed at the same tax rate, which implies that our tax system displays progressivity. Risk takers are only 2% of the population but pay 11% of the income taxes since they earn 11% of the income. We allow for mortgage interest rate deductibility. Because it is not possible to separate out interest rate from scheduled and unscheduled principal payments with our geometric mortgages, we assume that the entire mortgage payment is deductible but at a lower rate, $0.65 \times \tau$.[19]

We can interpret the risk-taker borrowing constraint parameters, ξ_G and ξ_P as regulatory capital constraints set by the government. Under Basel II and III, "first liens on a single-family home that are prudently underwritten and performing" enjoy a 50% risk weight and all others a 100% risk weight. Agency MBS receive a 20% risk weight. Given that we think of the non-guaranteed mortgage market as the subprime and Alt-A market, a capital charge of 8% seems most appropriate for ξ_P. Given that the government guaranteed mortgages are the counterpart to agency MBS, we set a capital charge of 1.6% for ξ_G.

Preference parameters We set the elasticity of inter-temporal substitution equal to 1 for all agents. The coefficients of risk aversion are $\sigma_R = 4$, $\sigma_B = 8$, and $\sigma_D = 30$. The annual subjective time discount factors are $\beta_R = \beta_D = 0.975$ and $\beta_B = 0.88$. Risk aversion and the time discount factor of the depositor are set such that we match the mean one-year risk-free interest rate of 1.6% and its volatility of 2.4%.[20] The borrower's discount factor is set to match the value of housing wealth to GDP over 1985-2014, which is 1.51 (Flow

expenditures (line 18) plus subsidies (line 27) minus the surplus of government enterprises (line 16). It excludes interest service on the debt and net spending on social security and other entitlement programs. Government revenues are defined as current receipts (line 1). Trend GDP is calculated with the Hodrick Prescott Filter.

[19] As discussed in Appendix B.3, the sum of all mortgage payments is $1/(1-\delta)$ and $F = \alpha/(1-\delta)$ is the payment of "principal." Hence, the fraction of "interest payments" is the fraction $(1-\alpha)/(1-\delta)$ which is 0.65.

[20] To calculate the real rate, we take the nominal one year constant maturity Treasury yield (FRED) and subtract expected inflation over the next 12 months. To form expected inflation, we fit an ARMA(1,1) model through monthly data of 12-month inflation over the period 1973-2014. The average rate and its volatility are computed over 1985-2014, just like most other targets.

of Funds). Borrower risk aversion is set to target the volatility of the annual change in household mortgage debt to GDP (Flow of Funds and NIPA), which is 4.2% in the 1985-2014 data. The risk taker subject discount factor is set equal to that of the other savers, the depositors. Their risk aversion is chosen to match average leverage ratios of the financial sector. Since mortgage assets are predominantly held by leveraged financial institutions, we calculate leverage for those kinds of institutions. The average ratio of total debt to total assets for 1985-2014 is 95.6%.[21]

Utility cost of risk-taker bankruptcy The model features a random utility penalty that risk takers suffer when they default. Because random default is mostly a technical assumption, it is sufficient to have a small penalty. We assume ρ_t is normally distributed with a mean of μ_ρ 1, i.e., a zero utility penalty on average, and a small standard deviation of $\sigma_\rho = 0.01$. The mean size of the penalty affects the frequency of financial sector defaults (and government bailouts). The lower μ_ρ, the lower the resistance to declare bankruptcy, and the higher the frequency of bank defaults. The standard deviation affects the correlation between negative financial intermediary wealth and bank defaults. Given those parameters, the frequency of financial crises (government bailouts of the risk-taker) depends on the frequency of foreclosure crises, and the endogenous choices (asset composition and liability choice) of the risk taker.

[21] Specifically, we include U.S. Chartered Commercial Banks and Savings Institutions, Foreign Banking offices in U.S., Bank Holding Companies, Banks in U.S. Affiliated Areas, Credit Unions, Finance Companies, Security Brokers and Dealers, Funding Corporations (Fed Bailout entities e.g. Maiden Lanes), GSEs, Agency- and GSE-backed Mortgage pools (before consolidation), Issuers of ABS, REITs, and Life and Property-Casualty Insurance Companies. Krisnamurthy and Vissing-Jorgensen (2011) identify a group of financial institutions as net suppliers of safe, liquid assets. This group is the same as ours except that we add insurance companies and take our money market mutual funds, since we are interested in leveraged financial firms. For comparison, leverage for the Krisnamurthy and Vissing-Jorgensen institutions is 90.7% for the 1985-2014 sample. The group of excluded, non-levered financial institutions are Money Market Mutual Funds, other Mutual Funds, Closed-end funds and ETFs, and State, Local, Federal, and Private Pension Funds. Total financial sector leverage, including these non-levered institutions, is 60.6%.

4 Main Results: Phasing out the GSEs

The main experiment in the paper is to compare an economy with and without government-guaranteed mortgages. More precisely, we compute a sequence of economies that differ by the guarantee fee γ_t that the government charges for providing the default insurance. All economies feature a government bailout guarantee to the financial sector (risk takers), or equivalently, deposit insurance. Higher g-fees "crowd-in" the private sector, a key objective of any reform proposal. We evaluate how equilibrium prices and quantities, and ultimately welfare are affected from an increase in g-fees. We do so based on a long simulation of the model. Table 2 summarizes the results.

4.1 Zero g-fees

We start with a model where the government provides the mortgage guarantee for free; $\gamma_t = 0$. Unsurprisingly, risk takers choose to obtain default insurance on all mortgages they issue. That is, they only hold guaranteed MBS, both in normal times (low σ_ω states) and in housing crises (high σ_ω states).

Risk takers finance these guaranteed MBS holdings in equilibrium by issuing deposits and combining them with their own equity capital contributions. By lending in long-term guaranteed MBS and borrowing short-term, banks bear mostly interest rate risk. They perform the traditional role of liquidity transformation.[22] Because banks have a modest risk aversion, compared with the much higher risk aversion of the depositors, banks are willing to provide safe assets to the depositors. They can do so at attractive rates, paying only 1.7% interest on deposits. Bearing no default risk on their assets, banks use substantial leverage in order to achieve their desired risk-return combination. The average bank leverage (book

[22] Guaranteed bonds in the model do suffer some losses when there is a default, mimicking what happens in reality to agency MBS. In particular, defaults act as prepayments at par. Since agency bonds usually trade above par prior to the prepayment, the prepayment constitutes a loss for the holder or agency MBS. In addition, prepayments happen when interest rates are low and reinvestment opportunities are poor. Mortgage REITS investing in agency MBS play a similar role to banks in reality. Some hybrid mortgage REITS combine agency and non-agency (private-label) MBS.

value of debt to market value of assets) ratio is 96.5%. Given that the margin requirement on guaranteed mortgage bonds is 1.6%, banks go almost all the way to their leverage limit. Specifically, the leverage constraint binds in 2/3 of the periods. Conversely, average risk taker wealth is small, only 1.8% of trend GDP. Banks have little "skin in the game." Due to low risk taker wealth and high leverage, the banking system is fragile. When adverse income or house value shocks hit, risk taker net worth threatens to fall below zero and the government steps in to bail our the financial sector. Such financial crises happen in 9% of the periods. In housing crises (high σ_ω periods), the frequency of financial crises is 28%. Thus housing crises are likely to trigger financial crises, as documented in the empirical work of Jorda et al. (2014). The average equilibrium return on the risk taker's wealth, the ROE of the banking sector, is 3.6% with an enormous volatility. The volatility is large because wealth frequently comes close to zero as a result of high leverage, leading to some very high returns, and because the return on wealth is −100% during risk taker bankruptcies.

Turning to the borrowers, we find that mortgage rates are low in this economy. The average mortgage rate is 3.2%, only 150bps above the deposit rate. Borrowers enjoy low rates in part due to the subsidized mortgage guarantees. Simultaneously house prices are high. Faced with high house prices and low mortgage rates, borrowers take out a lot of mortgage debt in equilibrium. The steady state stock of mortgages outstanding is high (0.334 market value or 0.027 units A^B). The average borrower LTV ratio is 78.7% and borrowers' mortgage debt-to-income is 1.44 on average, both are close to the averages in the recent SCF data. The average loss rate from mortgage default is 2.0%, with losses of 8.7% in housing crises and 1.3% in normal times. The volatility of mortgage originations, our measure of the stability of the provision of mortgage credit, is 2.7%.

Depositors have a strong precautionary savings demand for risk-free assets, given their high risk aversion. They are happy to lend to the banks at low interest rates because of their risk aversion and because the deposit insurance makes their claims on the banks risk free. In equilibrium, the entire supply of safe assets is provided by the banks because the government is a net lender. This itself is a consequence of the government raising more revenue from taxes than it spends on exogenous expenditures, debt service, and financial sector bailouts (deposit insurance fund payouts). The average net government debt is -2.7% of trend GDP. Thus the government and the depositors compete for risk-free assets issued by the banking sector. During housing crises, the depositor's precautionary savings incentives strengthen substantially and she desires to hold more risk-free assets. In those times, the government faces more frequent financial sector bailouts, which cause a deficit and lower its net position in risk-free debt to -1.1% of trend GDP. The risk taker accommodates the net increase in demand for risk-free debt by taking in more deposits. Risk taker leverage increases to 98.6% in these periods. The risk free interest rate falls precipitously to 0%.

In summary, the economy with zero g-fees features high house prices, high levels of mortgage debt, high mortgage debt to income ratios for borrowers, high rates of mortgage defaults and severities, high levels of bank debt, low levels of intermediary capital, frequent financial crises, higher government debt, and low interest rates.

4.2 Higher g-fees

The second and third economies we compute are cases where the g-fee is 15 basis points and 70 basis points on average, respectively. The 15 bps value reflects roughly what Fannie Mae and Freddie Mac charged the financial sector for mortgage insurance prior to 2009 while the 70 bps is higher than what Fannie and Freddie have been charging in the last couple of years.[23] Since the results are monotone in g-fees, we describe these two economies in parallel.

For the intermediate cost of mortgage default insurance, we find that risk-takers portfolio holdings depend on the state of the economy. During crisis times (high σ_ω states), banks hold almost only guaranteed mortgages, just like in the zero g-fee economy. However, during normal times, they hold almost exclusively private MBS. Put differently, banks only buy the government insurance in bad times. The state uncontingent g-fee is too cheap in bad times but too expensive in good times. This intermediate g-fee economy evaluates Option B of

[23]Back-of-the-envelope calculations by J.P. Morgan suggest that at g-fees of above 50bps, private execution dominates GSE execution for many loans (J.P. Morgan Securitized Products update 2014).

Table 2: Main Results: Unconditional Moments

	0 bp g-fee		15 bp g-fee		70 bp g-fee	
	mean	stdev	mean	stdev	mean	stdev
Prices						
Risk free rate	0.017	0.028	0.019	0.031	0.022	0.033
Mortgage rate	0.032	0.003	0.036	0.003	0.037	0.003
House price	1.761	0.090	1.694	0.091	1.669	0.094
Risk-Taker						
Market value of bank assets	0.334	0.014	0.304	0.007	0.295	0.007
Market value of guaranteed bonds	0.000	0.000	0.270	0.093	0.295	0.007
Market value of private bonds	0.334	0.014	0.034	0.093	0.000	0.001
Risk taker leverage	0.965	0.038	0.887	0.049	0.877	0.044
Risk taker wealth	0.018	0.013	0.041	0.015	0.043	0.013
Fraction λ^R binds	0.680	0.466	0.340	0.474	0.316	0.465
Bankruptcy frequency	0.094	0.292	0.008	0.086	0.000	0.000
Return on RT wealth[a]	0.022	0.756	0.036	0.274	0.041	0.255
Excess return on guaranteed bond	0.002	0.030	-0.009	0.030	-0.075	0.029
Excess return on private bond	-0.005	0.032	0.002	0.033	0.002	0.034
Borrower						
Mortgage debt	0.027	0.001	0.026	0.000	0.026	0.000
Borrower LTV	0.787	0.041	0.787	0.041	0.787	0.043
Market value of debt LTV	0.491	0.043	0.467	0.039	0.460	0.039
Borrower debt to income	1.444	0.031	1.389	0.023	1.368	0.024
Debt/income growth	0.000	0.027	0.000	0.027	0.000	0.029
Default rate	0.035	0.039	0.027	0.034	0.025	0.032
Loss Given Default	0.570	0.018	0.569	0.018	0.569	0.018
Loss rate private	0.020	0.024	0.016	0.021	0.015	0.020
Loss rate guaranteed	0.013	0.015	0.009	0.012	0.008	0.011
Loss rate portfolio	0.013	0.015	0.014	0.017	0.015	0.020
Government						
Government debt / GDP	-0.027	0.011	-0.033	0.002	-0.034	0.001
Welfare						
Aggregate Welfare	0.288	0.007	0.361	0.008	0.383	0.009
Deadweight Loss from Foreclosure	-0.005	0.006	-0.004	0.004	-0.003	0.004
Consumption borrower	0.297	0.025	0.296	0.024	0.295	0.024
Consumption depositor	0.408	0.015	0.408	0.012	0.409	0.011
Consumption risk taker	0.078	0.008	0.078	0.004	0.078	0.004

The table reports unconditional means and standard deviations of the main outcome variables from a 10,000 period simulation of three different models. The model in the first 2 columns has a mortgage guarantee fee of 0 (0 bp g-fee), the model in columns 3 and 4 has an average g-fee of 0.15% (expressed as a rate), and the model in the last two columns has an average g-fee of 0.70%.

[a]: Return on wealth is the return on the risk takers total portfolio i.e. their positive position in mortgages and negative position in deposits. It is set to -100% in periods, when risk takers declare bankruptcy or when risk takers wealth falls below 0%, and to 0% when initial risk taker wealth is negative. The series is winsorised at the 99.5th percentile.

the Obama Administration housing reform plan to set g-fees high enough that they are only attractive in crises.

In the high g-fee economy, risk takers shift exclusively towards holding private MBS. They do not buy default insurance from the government, neither in good nor in bad times. The g-fee is high enough to "crowd-in" the private sector at all times. The high g-fee economy evaluates Option A in the Obama plan which envisions an entirely private mortgage market.

A key result is that increasing the g-fee lowers the riskiness of the financial sector. Risk taker leverage in the intermediate g-fee economy averages 88.3% compared to 96.5% in the zero g-fee economy. It falls further to 87.7% in the high g-fee economy. This results is not just driven by the fact that private mortgages carry higher regulatory capital requirements ($\xi_P > \xi_G$). Indeed, the banks' leverage constraint binds in only about 30% of the periods, suggesting that banks choose to stay away from their leverage constraint in most periods. Since banks' portfolios now bear a substantial amount of mortgage default risk, there is less need to lever up in order to achieve the desired risk-return relationship for intermediary wealth. The return on risk-taker wealth, the banks' ROE, *increases* as g-fees rise: from 2.2% to 3.6% to 4.1%. This happens even as the amount of risk-taker wealth (intermediary capital) rises from 1.8% of GDP to 4.1% to 4.3%. Despite being better capitalized and having lower leverage, they earn higher returns on equity on average. The reason is that banks go bankrupt much less frequently as g-fees increase.

Borrowers indeed face higher interest rates. The mortgage rate increases from 3.2% in the zero g-fee economy to 3.6% in the intermediate case and 3.7% in the high g-fee economy. The government guarantee props up house price levels. House prices are 4% lower in the intermediate g-fee and 5.3% lower in the high g-fee economy than in the zero g-fee case. In equilibrium, because of lower house prices and higher mortgage rates, there is less mortgage debt outstanding and borrowers have less mortgage debt relative to their income. Borrower debt to income ratios fall from 144.4% in the zero g-fee economy to 138.9% in the intermediate g-fee and 136.8% in the high g-fee economy. Because of lower house prices, loan-to-value ratios are almost identical across economies (78.6%).

The lower indebtedness of the borrowers results in fewer defaults and hence in smaller loss rates for banks. The loss rate falls from 2.0% to 1.6% to 1.5% as we raise the g-fee. This decline is largely driven by smaller default rates during housing crises (13.8%, 12.0%, 11.3%) and smaller severities. In other words, absent government guarantees, banks not only reduce their leverage, they also take less risk on their mortgage portfolio. The safer intermediary balance sheet (higher equity, lower leverage) reduces the frequency of financial crises, or government bailouts dramatically from 9.4% in the zero g-fee economy to 0.8% in the intermediate economy and to 0 (in ten thousand simulations) in the high g-fee economy. Even though housing crises are just as frequent in all three economies by design (low σ_ω states happen with the same probability), they are not as severe in terms of losses the banking sector suffers, and combined with lower leverage and larger equity stakes, they do not result in financial crises. In the zero g-fee economy, 28% of housing crises lead to financial crises while in the intermediate economy only 2.4% do, and in the high g-fee economy none do. Thus, we conclude that it is the presence of the GSEs and the incentives the mortgage guarantees create for banks to lever up and make riskier mortgages that cause housing crises to result in systemic financial crises.

The increased stability of the financial system allows banks to expand mortgage credit in bad times (housing crises) in the high g-fee economies more than in the zero g-fee economy. The overall volatility in mortgage originations, as measured by the growth rate of mortgage debt/income, is quite similar in all three economies: 2.74%, 2.73%, and 2.90%. The popular fear that a private mortgage system would lead to large swings in the availability of mortgage credit, especially in bad times, is unwarranted in our model.

Turning to depositors, we find that they enjoy higher average deposit interest rates in the lower g-fee economies. The equilibrium risk-free rate goes from 1.7% to 1.9% to 2.2%. This result is surprising at first blush. The supply of risk-free securities is lower as g-fees rise because the intermediary balance sheet is smaller and leverage is smaller. Furthermore, because bank bailouts are less frequent and there is little equilibrium mortgage default insurance provision, the government spends less which further reduces the supply of risk-free assets. All else equal, a lower supply of risk-free debt should lead to lower interest rates. However, the depositor's demand for risk-free assets falls in the high g-fee economies, more than offsetting the reduced supply. The reason is a weaker precautionary savings motive, as depositors' consumption is less volatile in the high g-fee economies. The intuition for this result is that the optimal allocation of risk between risk takers and depositors is distorted more frequently in the zero g-fee economy. When the g-fee is high, risk takers are well capitalized and they are infrequently constrained in their intermediation capacity. They bear all mortgage default risk. In the zero g-fee economy, mortgage crisis episodes lead to frequent bankruptcies of risk takers, impeding their intermediation function. During these crises the risk free rate drops sharply, effectively making depositors bear a greater part of the mortgage default risk.

Finally, in terms of the government sector, the models with higher g-fees have lower government debt and less volatile government debt-to-GDP ratios (0.22% and 0.14% volatility compared to 1.00% in the zero g-fee world). They avoid an increase in debt in housing crises because there are much fewer/no payouts on mortgage guarantees and much less frequent/no financial sector bailouts. Here, all the effects of higher debt go through the interest rate. Despite higher government debt, the low g-fee economy has lower interest rates because the allocation of aggregate risk is worse.

4.3 Welfare

In the presence of deadweight costs of foreclosures, the lower losses on mortgages lead to an efficiency gain for the entire economy. The deadweight loss is 0.5% of GDP in the zero g-fee economy, but only 0.35% and 0.3% of GDP in the intermediate and high g-fee economies, respectively. The lower deadweight costs leave more resources for private consumption each period. A second source of welfare gains is that risk sharing between the different types of agents improves as g-fees increase. To measure the extent of the improvement, we compute the ratios of marginal utilities between the different types. If markets were complete, agents

Table 3: Welfare and Risk Sharing

	0 bp g-fee		15 bp g-fee		70 bp g-fee	
	mean	stdev	mean	stdev	mean	stdev
MU ratio borrower/risk taker[a]	-1.034	0.165	-1.782	0.113	-1.952	0.104
MU ratio risk taker/depositor	1.594	0.122	1.938	0.069	2.017	0.060
Value function borrower[b]	0.325	0.009	0.326	0.009	0.327	0.009
Value Function depositor	0.256	0.005	0.387	0.008	0.426	0.008
Value function risk taker	0.240	0.006	0.513	0.013	0.612	0.016

The table reports unconditional means and standard deviations from a 10,000 period simulation of three different models. The model in the first 2 columns has a mortgage guarantee fee of 0 (0 bp g-fee), the model in columns 3 and 4 has an average g-fee of 0.15% (expressed as a rate), and the model in the last two columns has an average g-fee of 0.70%.

[a]: Marginal utility ratios are calculated as the difference of the logarithm of marginal utilities.

[b]: With unit EIS the value functions are in units of composite consumption $C^{1-\rho}K^{\rho}$. Therefore differences in values have a direct interpretation as consumption-equivalent welfare differences.

would be able to achieve perfect risk sharing by forming portfolios that keep these ratios constant. Hence larger volatilities of these marginal utility (MU) ratios indicate worse risk sharing between the different types of agents.

Table 3 lists the average MU ratios and their volatilities for borrowers/risk takers, and risk takers/depositors, as these are the pairs of agents that directly trade with each other. The volatilities of both ratios decrease as g-fees rise, indicating an improvement in risk sharing . The presence of cheap mortgage guarantees in effect leads to more fragile financial intermediaries (risk takers). This in turn means that both the flow of funds from patient savers to impatient borrowers, and the risk allocation among savers of different risk tolerance break down more frequently.

Tables 2 and 3 show the increase in welfare due to improved risk sharing and fewer deadweight losses. Raising g-fees from zero to the intermediate value increases social welfare by 25%. Raising the g-fee further increases welfare by another 5% for a total gain of 30%. All three agents are better off. The borrower's welfare increases the least, by 0.6% in the full public-to-private transition. The absence of a loss in welfare for borrowers is surprising since providing subsidized mortgage guarantees obviously directly benefits borrowers and taking away these subsidies hurts them. Despite the higher mortgage rates, borrowers benefit substantially from the improved risk sharing in the private economy. Depositor's welfare increases by 66% while risk taker welfare increases by a factor of 2.5 (consistent with the fact that their average wealth more than doubles). The reform redistributes wealth to the risk takers/bankers, raising inequality. But it makes everyone, and thus the economy as a whole, better off.

One rationale for mortgage guarantees is the stability in the provision of mortgages due to the government backstop. Taken as given a highly levered balance sheet of banks and the riskiness of the mortgages they originate, a shock to output (for example) could decimate the banks in the absence of guarantees and lead them to sharply reduce new mortgage originations. With guarantees, the government would step in and use its ability to issue debt to smooth out the negative shock over time. Banks would not be hurt nearly as much because of the mortgage default insurance they enjoy. This intertemporal smoothing technology of the government is operational in our model and does mitigate the effects of the shock. But the logic ignores the endogenous response of the banks' leverage and mortgage origination choice. Absent guarantees, banks become more prudent which benefits the stability of mortgage origination and of the financial system. In our model economy, the private sector turns out to be able to deliver superior consumption smoothing and risk sharing outcomes.

While absent in the model, if income taxation were distortionary and some of the higher spending in the zero g-fee economy due to the dual guarantee had to be raised from taxes, such distortions would further amplify the welfare costs of low g-fees. Similarly, if financial crises had negative effects on other unmodeled sectors of the economy, that would further amplify the welfare effects as well.

5 Conclusion

Our main findings are that underpriced, government-provided mortgage default insurance distorts the incentives of the financial sector in a way that leads it to take more risk in the mortgages it originates and in the leverage it takes on. While the policy leads to higher house prices, low mortgage rates, and low interest rates, it also leads to more frequent mortgage defaults, and financial crises when banks become insolvent. Our model thus highlights the connection between housing market risk and financial sector risk, a prominent feature of financial systems in developed economies. While the government can mitigate the fallout from such crises by spreading the costs out over time via the issuance of government debt, the ultimate allocation of risk remains suboptimal. We document large welfare gains from transitioning to a private mortgage system, a transition which can be effectuated by raising the cost of the government mortgage guarantees. A safer financial sector emerges which is better able to intermediate between borrowers and savers so as to implement the optimal allocation of risk in the economy. While all agents in the economy gain from the transition, the bankers gain the most so that the policy increases wealth inequality.

More broadly, the paper brings together the literatures of financial intermediary-based asset pricing and housing finance reform. It brings in the role of the government into the former and the importance of the financial sector into the latter. New is the possibility of default and the government's provision of bailout guarantees to the creditors' of the banking sector, as well as the interaction of those guarantees on banks' liabilities with the mortgage guarantees on banks' assets.

Our model is a natural laboratory to explore the effects of government purchases of whole mortgage loans, guaranteed mortgage bonds, and private mortgage bonds. The GSEs were a large buyer in all categories accumulating a combined portfolio of $1.7 trillion dollars by 2007. Equally interesting would be to study the impact of the subsequent 50% reduction of that balance sheet to $850 billion at the end of 2014. Similarly, the Federal Reserve was a large buyer of guaranteed mortgage bonds, accumulating $1.8 trillion as part of its QE1 and

QE3 programs. While current policy is to keep the size of this portfolio constant, studying the change to a no-reinvestment policy would be interesting. Over the next several years, we are likely to see a change from governmental to private sector ownership of 25% of one of the largest fixed income markets in the world. A complete understanding of the impact of these large purchases or sales on the mortgage market, house prices, the macro-economy, and the financial sector remains an important challenge for future research.

There are several other promising avenues for further exploration. The model currently abstracts from the choice between owning and renting. Abolishing the mortgage guarantees may well affect the home ownership rate. If house price-to-rent ratios fall in the aftermath of the policy reform, as they do in recent models that study the abolition of mortgage interest rate deductibility, they may well boost home ownership rates. A second ingredient our work abstracts from is the feedback effect from the mortgage lending complex to the rest of the financial sector and to the real economy. In a world with subsidized mortgage lending, lending to non-financial businesses with good ideas gets crowded out. Future work could add a group of capital-constrained entrepreneurs with productive investment opportunities, thereby endogenizing the exogenous endowment process considered here. Such a model would add a further, possibly longer-term cost of government mortgage guarantees and financial sector bailout guarantees.

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A Appendix A: Model Solution

We reformulate the problem of risk taker, depositor, and borrower to ensure stationarity of the problem. We do so by scaling all variables by permanent income.

A.1 Borrower

A.1.1 Preliminaries

We start by defining some preliminaries.

$$
\begin{aligned}
Z_A(\omega_t^*) &= [1 - F_\omega(\omega_t^*;\chi)] \\
Z_K(\omega_t^*) &= [1 - F_\omega(\omega_t^*;\chi)]\mathrm{E}\left[w_{i,t} \mid \omega_{i,t} \geq \omega_t^*;\chi\right]
\end{aligned}
$$

and $F_\omega(\cdot;\chi)$ is the CDF of $\omega_{i,t}$ with parameters χ. Assume $\omega_{i,t}$ are drawn from a Gamma distribution with shape and scale parameters $\chi = (\chi_0, \chi_1)$ such that

$$
\begin{aligned}
\mu_\omega &= \mathrm{E}_i[\omega_{i,t};\chi_0,\chi_1] = \chi_0\chi_1 \\
\sigma^2_{t,\omega} &= \mathrm{Var}_i[\omega_{i,t};\chi_0,\chi_1] = \chi_0\chi_1^2
\end{aligned}
$$

From Landsman and Valdez (2004, equation 22), we know that

$$
\mathrm{E}[\omega|\omega \geq \bar{\omega}] = \mu_\omega \frac{1 - F_\omega(\bar{\omega};\chi_0+1,\chi_1)}{1 - F_\omega(\bar{\omega};\chi_0,\chi_1)}
$$

so the closed form expression for Z_K is

$$
Z_K(\omega_t^*) = \mu_\omega\left[1 - F_\omega(\omega_t^*;\chi_0+1,\chi_1)\right]
$$

It is useful to compute the derivatives of $Z_K(\cdot)$ and $Z_A(\cdot)$:

$$
\begin{aligned}
\frac{\partial Z_K(\omega_t^*)}{\partial \omega_t^*} &= \frac{\partial}{\partial \omega_t^*}\int_{\omega_t^*}^{\infty} \omega f_\omega(\omega)d\omega = -\omega_t^* f_\omega(\omega_t^*), \\
\frac{\partial Z_A(\omega_t^*)}{\partial \omega_t^*} &= \frac{\partial}{\partial \omega_t^*}\int_{\omega_t^*}^{\infty} f_\omega(\omega)d\omega = -f_\omega(\omega_t^*),
\end{aligned}
$$

where $f_\omega(\cdot)$ is the p.d.f. of a Gamma distribution with parameters (χ_0, χ_1).

A.1.2 Statement of stationary problem

Let $\mathcal{S}_t^B = \left(g_t, \sigma_{\omega,t}, W_t^R, W_t^D, B_{t-1}^G\right)$ represent state variables exogenous to the borrower's decision. We consider the borrower's problem in the current period after income and house depreciation shocks have been realized, after the risk taker has chosen a default policy, and after the random utility penalty is realized. Then the borrower's value function, transformed to ensure stationarity, is:

$$
\begin{aligned}
V^B(K_{t-1}^B, A_t^B, \mathcal{S}_t^B) = \max_{\{C_t^B, K_t^B, \omega_t^*, B_t^B\}} \Bigg\{ &(1-\beta_B)\left[\left(C_t^B\right)^{1-\theta}\left(A_K K_{t-1}^B\right)^{\theta}\right]^{1-1/\nu} + \\
&+ \beta_B \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^B(K_t^B, A_{t+1}^B, \mathcal{S}_{t+1}^B)\right)^{1-\sigma_B}\right]^{\frac{1-1/\nu}{1-\sigma_B}} \Bigg\}^{\frac{1}{1-1/\nu}}
\end{aligned}
$$

subject to

$$C_t^B = (1-\tau_t^B)Y_t^B + Z_K(\omega_t^*)p_t K_{t-1}^B + q_t^m B_t^B - (1-\tau_t^m)Z_A(\omega_t^*)A_t^B - p_t K_t^B \tag{16}$$

$$A_{t+1}^B = e^{-g_{t+1}}\left[\delta Z_A\left(\omega_t^*\right)A_t^B + B_t^B\right] \tag{17}$$

$$\phi_t p_t K_t^B \geq \frac{1}{1-\delta}\left[\delta Z_A\left(\omega_t^*\right)A_t^B + B_t^B\right] \tag{18}$$

$$\mathcal{S}_{t+1}^B = h(\mathcal{S}_t^B) \tag{19}$$

where the functions Z_K and Z_A are defined in the preliminaries above.

The continuation value $\tilde{V}^B(\cdot)$ must take into account the default decision of the risk taker at the beginning of next period. We anticipate here and show below that the default decision takes the form of a cutoff rule:

$$\begin{aligned}\tilde{V}^B\left(K_{t-1}^B, A_t^B, \mathcal{S}_t^B\right) =& F_\rho(\rho_t^*)\mathrm{E}_\rho\left[V^B(K_{t-1}^B, A_t^B, \mathcal{S}_t^B)\,|\,\rho < \rho_t^*\right] + (1-F_\rho(\rho_t^*))\,\mathrm{E}_\rho\left[V^B(K_{t-1}^B, A_t^B, \mathcal{S}_t^B)\,|\,\rho > \rho_t^*\right] \\ =& F_\rho(\rho_t^*)V^S(K_{t-1}^B, A_t^B, \mathcal{S}_t^S(\rho_t < \rho_t^*)) + (1-F_\rho(\rho_t^*))\,V^S(K_{t-1}^B, A_t^B, \mathcal{S}_t^S(\rho_t > \rho_t^*)),\end{aligned} \tag{20}$$

where (20) obtains because the expectation terms conditional on realizations of ρ_t and ρ_t^* only differ in the values of the aggregate state variables.

Denote the value function and the partial derivatives of the value function as:

$$V_t^B \equiv V(K_{t-1}^B, A_t^B, \mathcal{S}_t^B),$$
$$V_{A,t}^B \equiv \frac{\partial V(K_{t-1}^B, A_t^B, \mathcal{S}_t^B)}{\partial A_t^B},$$
$$V_{K,t}^B \equiv \frac{\partial V(K_{t-1}^B, A_t^B, \mathcal{S}_t^B)}{\partial K_{t-1}^B}.$$

Therefore the marginal values of borrowing and of housing of $\tilde{V}^B(\cdot)$ are:

$$\tilde{V}_{A,t}^B = F_\rho(\rho_t^*)\frac{\partial V^B(K_{t-1}^B, A_t^B, \mathcal{S}_t^B(\rho_t < \rho_t^*))}{\partial A_t^B} + (1-F_\rho(\rho_t^*))\,\frac{\partial V^B(K_{t-1}^B, A_t^B, \mathcal{S}_t^B(\rho_t > \rho_t^*)}{\partial A_t^B}$$
$$\tilde{V}_{K,t}^B = F_\rho(\rho_t^*)\frac{\partial V^B(K_{t-1}^B, A_t^B, \mathcal{S}_t^B(\rho_t < \rho_t^*))}{\partial K_{t-1}^B} + (1-F_\rho(\rho_t^*))\,\frac{\partial V^B(K_{t-1}^B, A_t^B, \mathcal{S}_t^B(\rho_t > \rho_t^*)}{\partial K_{t-1}^B}$$

Denote the certainty equivalent of future utility as:

$$CE_t^B = \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^B(K_t^B, A_{t+1}^B, \mathcal{S}_{t+1}^B)\right)^{1-\sigma_B}\right]^{\frac{1}{1-\sigma_B}}$$

Recall that

$$u_t^B = \left(C_t^B\right)^{1-\theta}\left(A_K K_{t-1}^B\right)^\theta$$

A.1.3 First-order conditions

New mortgages The FOC for new mortgage loans B_t^B is:

$$0 = \frac{1}{1-1/\nu}\left\{(1-\beta_B)\left[\left(C_t^B\right)^{1-\theta}\left(A_K K_{t-1}^B\right)^{\theta}\right]^{1-1/\nu} + \right.$$

$$\left. + \beta_B \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^B(K_t^B, A_{t+1}^B, \mathcal{S}_{t+1}^B)\right)^{1-\sigma_B}\right]^{\frac{1-1/\nu}{1-\sigma_B}}\right\}^{\frac{1}{1-1/\nu}-1} \times$$

$$\times \left\{(1-1/\nu)(1-\beta_B)\left[\left(C_t^B\right)^{1-\theta}\left(A_K K_{t-1}^B\right)^{\theta}\right]^{-1/\nu}(1-\theta)(A_K K_{t-1}^B)^{\theta}(C_t^B)^{-\theta} q_t^m + \right.$$

$$+ \beta_B \frac{1-1/\nu}{1-\sigma_B}\mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^B(K_t^B, A_{t+1}^B, \mathcal{S}_{t+1}^B)\right)^{1-\sigma_B}\right]^{\frac{1-1/\nu}{1-\sigma_B}-1} \times$$

$$\left. \times\ E_t\left[(1-\sigma_B)\left(e^{g_{t+1}}\tilde{V}^B(K_t^B, A_{t+1}^B, \mathcal{S}_{t+1}^B)\right)^{-\sigma_B} e^{g_{t+1}}\tilde{V}_{A,t+1}^B e^{-g_{t+1}}\right]\right\} - \lambda_t^B \frac{1}{1-\delta}$$

where λ_t^B is the Lagrange multiplier on the borrowing constraint.

Simplifying, we get:

$$q_t^m \frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu} = \\ \lambda_t^B \frac{1}{1-\delta} - \beta_B \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^B)^{-\sigma_B}\tilde{V}_{A,t+1}^B](CE_t^B)^{\sigma_B - 1/\nu}(V_t^B)^{1/\nu} \tag{21}$$

Houses The FOC for new purchases of houses K_t^B is:

$$0 = \frac{1}{1-1/\nu}(V_t^B)^{1/\nu} \times \left\{-(1-1/\nu)(1-\beta_B)(u_t^B)^{-1/\nu}(1-\theta)(A_K K_{t-1}^B)^{\theta}(C_t^B)^{-\theta} p_t + \right.$$

$$\left. + \frac{1-1/\nu}{1-\sigma_B}\beta_B (CE_t^B)^{\sigma_B - 1/\nu}\mathrm{E}_t[(1-\sigma_B)(e^{g_{t+1}}\tilde{V}_{t+1}^B)^{-\sigma_B} e^{g_{t+1}}\tilde{V}_{K,t+1}^B]\right\} + \lambda_t^B \phi_t p_t.$$

Simplifying, we get:

$$p_t \frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu} = \\ \lambda_t^B \phi_t p_t + \beta_B \mathrm{E}_t[e^{(1-\sigma_B)g_{t+1}}(\tilde{V}_{t+1}^B)^{-\sigma_B}\tilde{V}_{K,t+1}^B](CE_t^B)^{\sigma_B - 1/\nu}(V_t^B)^{1/\nu} \tag{22}$$

Default Threshold Taking the first-order condition with respect to $\omega_t^\star$ and using the expressions for the derivatives of $Z_K(\cdot)$ and $Z_A(\cdot)$ in the preliminaries above yields:

$$f_\omega(\omega_t^*)\left[\omega_t^* p_t K_{t-1}^B - (1-\tau_t^m)A_t^B\right]\frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu} = \\ \delta A_t^B f_\omega(\omega_t^*)\left\{\lambda_t^B \frac{1}{1-\delta} - \beta_B \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}_{t+1}^B\right)^{-\sigma_B}\tilde{V}_{A,t+1}^B\right] \times (CE_t^B)^{\sigma_B - 1/\nu}(V_t^B)^{1/\nu}\right\}.$$

This can be simplified by replacing the term in braces on the RHS using the FOC for new loans (21) and solving for ω_t^* to give:

$$\omega_t^* = \frac{A_t^B(1-\tau_t^m + \delta q_t^m)}{p_t K_{t-1}^B}. \tag{23}$$

Maximum Loan-to-Value Ratio The maximum LTV parameter ϕ_t in (18) is set so that lenders achieve the same expected loss on their mortgage portfolio each period. Let that constant loss rate be φ. Lenders adjust the maximum LTV ratio ϕ_t so that the expected loss when the constraint binds is equal to the target:

$$\mathrm{E}_t\left[L(\phi_t)\right] = \varphi,$$

where the loss rate (per unit of the bond) is:

$$L(\phi_t) \equiv 1 - \frac{(1-\zeta)\frac{(\mu_\omega - Z_K(\bar{\omega}_{t+1}))}{\phi_t} + (1+\delta q^m_{t+1})Z_A(\bar{\omega}_{t+1})}{1+\delta q^m_{t+1}},$$

and the associated default threshold at the beginning of next period when the constraint binds is

$$\bar{\omega}_{t+1} = \frac{(1-\tau^m_{t+1}+\delta q^m_{t+1})\phi_t p_{t+1} K^B_t}{p_{t+1}K^B_t} = \phi_t(1-\tau^m_{t+1}+\delta q^m_{t+1}).$$

Holding fixed q^m_{t+1}, a higher ϕ_t increases the loss rate because fewer mortgages will be repaid (Z_A is lower), and the recovery value of the foreclosed houses is lower.

A.1.4 Marginal Values of State Variables and SDF

Mortgages Taking the derivative of the value function with respect to A^B_t gives:

$$\begin{aligned} V^B_{A,t} =& -(1-\tau^m_t)Z_A(\omega^*_t)\frac{1-\theta}{C^B_t}(1-\beta_B)(V^B_t)^{1/\nu}(u^B_t)^{1-1/\nu} \\ & - \delta Z_A(\omega^*_t)\{\lambda^B_t\frac{1}{1-\delta} - \beta_B \mathrm{E}_t[e^{g_{t+1}}(e^{g_{t+1}}\tilde{V}^B_{t+1})^{-\sigma_B}\tilde{V}^B_{A,t+1}] \times (CE^B_t)^{\sigma_B-1/\nu}(V^B_t)^{1/\nu}\}. \end{aligned}$$

Note that we can substitute for the term in braces using equation (21).

$$V^B_{A,t} = -Z_A(\omega^*_t)\left(1-\tau^m_t+\delta q^m_t\right)\frac{1-\theta}{C^B_t}(1-\beta_B)(V^B_t)^{1/\nu}(u^B_t)^{1-1/\nu}. \tag{24}$$

Houses Taking the derivative of the value function with respect to K^B_{t-1} gives:

$$V^B_{K,t} = \left[p_t Z_K(\omega^*_t) + \frac{\theta C^B_t}{(1-\theta)K^B_{t-1}}\right]\frac{1-\theta}{C^B_t}(V^B_t)^{1/\nu}(1-\beta_B)(u^B_t)^{1-1/\nu}. \tag{25}$$

SDF Define the borrower's intertemporal marginal rate of substitution between t and $t+1$, conditional on a particular realization of ρ_{t+1} as:

$$\begin{aligned} \mathcal{M}^B_{t,t+1}(\rho_{t+1}) &= \frac{\partial V^B_t/\partial C^B_{t+1}}{\partial V^B_t/\partial C^B_t} = \frac{\partial V^B_t}{\partial V^B_{t+1}}e^{-g_{t+1}}\frac{\partial V^B_{t+1}/\partial C^B_{t+1}}{\partial V^B_t/\partial C^B_t} \\ &= (V^B_t)^{1/\nu}\beta_B(CE^B_t)^{\sigma_B-1/\nu}(e^{g_{t+1}}\tilde{V}^B_{t+1})^{-\sigma_B}\frac{\frac{1-\theta}{C^B_{t+1}}(1-\beta_B)(V^B_{t+1})^{1/\nu}(u^B_{t+1})^{1-1/\nu}}{\frac{1-\theta}{C^B_t}(1-\beta_B)(V^B_t)^{1/\nu}(u^B_t)^{1-1/\nu}} \\ &= \beta_B e^{-\sigma_B g_{t+1}}\left(\frac{C^B_{t+1}}{C^B_t}\right)^{-1}\left(\frac{u^B_{t+1}}{u^B_t}\right)^{1-1/\nu}\left(\frac{V^B_{t+1}}{CE^B_t}\right)^{-(\sigma_B-1/\nu)} \end{aligned}$$

We can then define the stochastic discount factor (SDF) of borrowers as:

$$\tilde{\mathcal{M}}^B_{t,t+1} = F_\rho(\rho^*_{t+1})\mathcal{M}^B_{t,t+1}(\rho_{t+1}<\rho^*_{t+1}) + (1-F_\rho(\rho^*_{t+1}))\mathcal{M}^B_{t,t+1}(\rho_{t+1}>\rho^*_{t+1}),$$

where $\mathcal{M}^B_{t,t+1}(\rho_{t+1} < \rho^*_{t+1})$ and $\mathcal{M}^B_{t,t+1}(\rho_{t+1} > \rho^*_{t+1})$ are the IMRSs, conditional on the two possible realizations of state variables.

A.1.5 Euler Equations

Mortgages Observe that we can rewrite equation (21) as:

$$q^m_t = \frac{C^B_t}{(1-\theta)(1-\beta_B)(V^B_t)^{1/\nu}(u^B_t)^{1-1/\nu}}\left\{\lambda^B_t \frac{1}{1-\delta} - \beta_B \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}^B_{t+1})^{-\sigma_B}\tilde{V}^B_{A,t+1}](CE^B_t)^{\sigma_B-1/\nu}(V^B_t)^{1/\nu}\right\}.$$

We define the rescaled Lagrange multiplier of the borrower as the original multiplier divided by marginal utility of consumption:

$$\tilde{\lambda}^B_t = \lambda^B_t \frac{C^B_t}{(1-\theta)(1-\beta_B)(V^B_t)^{1/\nu}(u^B_t)^{1-1/\nu}}.$$

Recall that $\tilde{V}^B_{A,t+1}$ is a linear combination of $V^B_{A,t+1}$ conditional on ρ_t being below and above the threshold, and with each $V^B_{A,t+1}$ given by equation (24). Substituting in for $\tilde{V}^B_{A,t+1}$ and using the SDF expression, we get the recursion:

$$q^m_t = \tilde{\lambda}^B_t \frac{1}{1-\delta} + \mathrm{E}_t\left[\tilde{\mathcal{M}}^B_{t,t+1} Z_A(\omega^*_{t+1})\left(1-\tau^B_{t+1}+\delta q^m_{t+1}\right)\right]. \tag{26}$$

Houses Likewise, observe that we can write (22) as:

$$p_t\left[1-\tilde{\lambda}^B_t\phi\right] = \frac{\beta_B \mathrm{E}_t[e^{g_{t+1}}(e^{g_{t+1}}\tilde{V}^B_{t+1})^{-\sigma_B}\tilde{V}^B_{K,t+1}](CE^B_t)^{\sigma_B-1/\nu}(V^B_t)^{1/\nu}}{\frac{1-\theta}{C^B_t}(1-\beta_B)(V^B_t)^{1/\nu}(u^B_t)^{1-1/\nu}}$$

Recall that $\tilde{V}^B_{K,t+1}$ is a linear combination of $V^B_{K,t+1}$ conditional on ρ_t being below and above the threshold, and with each $V^B_{K,t+1}$ given by equation (25). Substituting in for $\tilde{V}^B_{K,t+1}$ and using the SDF expression, we get the recursion:

$$p_t\left[1-\tilde{\lambda}^B_t\phi\right] = \mathrm{E}_t\left[\tilde{\mathcal{M}}^B_{t,t+1}e^{g_{t+1}}\left\{p_{t+1}Z_K(\omega^*_{t+1}) + \frac{\theta C^B_{t+1}}{(1-\theta)K^B_t}\right\}\right] \tag{27}$$

A.2 Depositor

We solve a restricted problem where we assume that the depositor does not buy private long-term mortgage bonds. After solving the model under this assumption, we go back and verify that the depositor indeed does not want to deviate from a zero position in equilibrium.

A.2.1 Statement of stationary problem

Let $\mathcal{S}^D_t = \left(g_t, \sigma_{\omega,t}, W^R_t, A^B_t, B^G_{t-1}\right)$ be the depositor's state vector capturing all exogenous state variables. Scaling by permanent income, the stationary problem of the depositor -after the risk taker has made default her decision and the utility cost of default is realized- is:

$$V^D(W^D_t,\mathcal{S}^D_t) = \max_{\{C^D_t, B^D_t, A^D_{t+1,G}\}}\left\{(1-\beta_D)\left[\left(C^D_t\right)^{1-\theta}\left(A_K K^D_{t-1}\right)^{\theta}\right]^{1-1/\nu} + \right.$$
$$\left. + \beta_D \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^D(W^D_{t+1},\mathcal{S}^D_{t+1})\right)^{1-\sigma_D}\right]^{\frac{1-1/\nu}{1-\sigma_D}}\right\}^{\frac{1}{1-1/\nu}}$$

subject to

$$C_t^D = (1-\tau_t^S)Y_t^D + W_t^D - (q_t^m + \gamma_t)A_{t+1,G}^D - q_t^f B_t^D - (1-\mu_{t,\omega})p_t K_{t-1}^D \tag{28}$$

$$W_{t+1}^D = e^{-g_{t+1}}\left[(M_{t+1,G} + \delta Z_A(\omega_{t+1}^*)q_{t+1}^m)A_{t+1,G}^D + B_t^D\right] \tag{29}$$

$$B_t^D \geq 0 \tag{30}$$

$$A_{t+1,G}^D \geq 0 \tag{31}$$

$$\mathcal{S}_{t+1}^D = h(\mathcal{S}_t^D) \tag{32}$$

As before, we will drop the arguments of the value function and denote marginal values of wealth and mortgages as:

$$V_t^D \equiv V_t^D(W_t^D, \mathcal{S}_t^D),$$
$$V_{W,t}^D \equiv \frac{\partial V_t^D(W_t^D, \mathcal{S}_t^D)}{\partial W_t^D},$$

Denote the certainty equivalent of future utility as:

$$CE_t^D = \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^D(W_t^D, \mathcal{S}_t^D)\right)^{1-\sigma_D}\right],$$

and the composite within-period utility as:

$$u_t^D = (C_t^D)^{1-\theta}(A_K K_{t-1}^D)^\theta.$$

Like the borrower, the depositor must take into account the risk-taker's default decisions and the realization of the utility penalty of default. Therefore the marginal value of wealth is:

$$\tilde{V}_{W,t}^D = F_\rho(\rho_t^*)\frac{\partial V^D(W_t^D, \mathcal{S}_t^D(\rho_t < \rho_t^*))}{\partial W_t^D} + (1 - F_\rho(\rho_t^*))\,\frac{\partial V^D(W_t^D, \mathcal{S}_t^D(\rho_t > \rho_t^*)}{\partial W_t^D}.$$

A.2.2 First-order conditions

The first-order condition for the short-term bond position is:

$$\begin{aligned}
&q_t^f \frac{1-\theta}{C_t^D}(1-\beta_D)(V_t^D)^{1/\nu}(u_t^D)^{1-1/\nu} = \\
&\lambda_t^D + \beta_D \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^D)^{-\sigma_D}\tilde{V}_{W,t+1}^D](CE_t^D)^{\sigma_D - 1/\nu}(V_t^D)^{1/\nu}
\end{aligned} \tag{33}$$

where λ_t^D is the Lagrange multiplier on the borrowing constraint (30).

The first order condition for the government-guaranteed mortgage bond position is:

$$\begin{aligned}
&(q_t^m + \gamma_t)\frac{1-\theta}{C_t^D}(1-\beta_D)(V_t^D)^{1/\nu}(u_t^D)^{1-1/\nu} = \\
&\mu_{G,t}^D + \beta_D \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^D)^{-\sigma_D}\tilde{V}_{W,t+1}^D\left(M_{G,t+1} + \delta Z_A(\omega_{t+1}^*)q_{t+1}^m\right)](CE_t^D)^{\sigma_D - 1/\nu}(V_t^D)^{1/\nu},
\end{aligned} \tag{34}$$

where $\mu_{t,G}^D$ is the Lagrange multiplier on the no-shorting constraint for guaranteed loans (31).

A.2.3 Marginal Values of State Variables and SDF

Marginal value of wealth is:

$$V_{W,t}^D = \frac{1-\theta}{C_t^D}(1-\beta_D)(V_t^D)^{1/\nu}(u_t^D)^{1-1/\nu}, \tag{35}$$

and for the continuation value function:

$$\tilde{V}_{W,t}^D = F_\rho(\rho_t^*)\frac{\partial V^D(W_t^D, \mathcal{S}_t^D(\rho_t < \rho_t^*))}{\partial W_t^D} + (1 - F_\rho(\rho_t^*))\,\frac{\partial V^D(W_t^D, \mathcal{S}_t^D(\rho_t > \rho_t^*)}{\partial W_t^D}.$$

Defining the SDF in the same fashion as we did for the borrower, we get:

$$\mathcal{M}_{t,t+1}^D(\rho_t) = \beta_D e^{-\sigma_D g_{t+1}} \left(\frac{V_{t+1}^D}{CE_t^D}\right)^{-(\sigma_D - 1/\nu)} \left(\frac{C_{t+1}^D}{C_t^D}\right)^{-1} \left(\frac{u_{t+1}^D}{u_t^D}\right)^{1-1/\nu},$$

and

$$\tilde{\mathcal{M}}_{t,t+1}^D = F_\rho(\rho_{t+1}^*)\mathcal{M}_{t,t+1}^D(\rho_{t+1} < \rho_{t+1}^*) + (1 - F_\rho(\rho_{t+1}^*))\mathcal{M}_{t,t+1}^D(\rho_{t+1} > \rho_{t+1}^*).$$

A.2.4 Euler Equations

Combining the first-order condition for short-term bonds (33) with the marginal value of wealth, and the SDF, we get the Euler equation for the short-term bond:

$$q_t^f = \tilde{\lambda}_t^D + \mathrm{E}_t\left[\tilde{\mathcal{M}}_{t,t+1}^D\right] \tag{36}$$

where $\tilde{\lambda}_t^D$ is the original multiplier λ_t^D divided by the marginal value of wealth.

Similarly, from (34) we get the Euler Equation for guaranteed mortgages:

$$q_t^m + \gamma_t = \tilde{\mu}_{G,t}^D + \mathrm{E}_t\left[\tilde{\mathcal{M}}_{t,t+1}^D\left(M_{G,t+1} + \delta Z_A(\omega_{t+1}^*)q_{t+1}^m\right)\right] \tag{37}$$

A.3 Risk Takers

A.3.1 Statement of stationary problem

Denote by W_t^R risk taker wealth at the beginning of the period, before their bankruptcy decision. Then wealth after realization of the penalty ρ_t is:

$$\tilde{W}_t^R = (1 - D(\rho_t))W_t^R,$$

and the effective utility penalty is:

$$\tilde{\rho}_t = D(\rho_t)\rho_t.$$

Let $\mathcal{S}_t^R = \left(g_t, \sigma_{\omega,t}, W_t^D, A_t^B, B_{t-1}^G\right)$ denote all other aggregate state variables exogenous to risk takers.

After the default decision, risk takers face the following optimization problem over consumption and portfolio composition, formulated to ensure stationarity:

$$V^R(\tilde{W}_t^R, \tilde{\rho}_t, \mathcal{S}_t^R) = \max_{C_t^R, A_{t+1,P}^R, A_{t+1,P}^R, B_t^R} \Bigg\{(1-\beta_R)\left[\frac{(C_t^R)^{1-\theta}(K_{t-1}^R)^\theta}{e^{\tilde{\rho}_t}}\right]^{1-1/\nu} + \beta_R \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^R\left(W_{t+1}^R, \mathcal{S}_{t+1}^R\right)\right)^{1-\sigma_R}\right]^{\frac{1-1/\nu}{1-\sigma_R}}\Bigg\}^{\frac{1}{1-1/\nu}} \tag{38}$$

subject to:

$$
\begin{aligned}
(1-\tau^S)Y_t^R + \tilde{W}_t^R =& C_t^R + (1-\mu_\omega)p_t K_{t-1}^R + q_t^m A_{t+1,P}^R + (q_t^m+\gamma_t)A_{t+1,G}^R + q_t^f B_t^R, & (39)\\
W_{t+1}^R =& e^{-g_{t+1}}\left[(M_{t+1,P}+\delta Z_A(\omega_{t+1}^*)q_{t+1}^m)A_{t+1,P}^R + (M_{t+1,G}+\delta Z_A(\omega_{t+1}^*)q_{t+1}^m)A_{t+1,G}^R + B_t^R\right], & (40)\\
B_t^R \geq& -q_t^m(\xi_P A_{t+1,P}^R + \xi_G A_{t+1,G}^R), & (41)\\
A_{t+1,G}^R \geq& 0, & (42)\\
A_{t+1,P}^R \geq& 0, & (43)\\
\mathcal{S}_{t+1}^R =& h(\mathcal{S}_t^R). & (44)
\end{aligned}
$$

The continuation value $\tilde{V}^R\left(W_{t+1}^R, \mathcal{S}_{t+1}^R\right)$ is the outcome of the optimization problem risk takers face at the beginning of the following period, i.e., before the decision over the optimal bankruptcy rule. This continuation value function is given by:

$$
\tilde{V}^R(W_t^R, \mathcal{S}_t^R) = \max_{D(\rho)} \mathrm{E}_\rho\left[D(\rho)V^R(0,\rho,\mathcal{S}_t^R) + (1-D(\rho))V^R(W_t^R,0,\mathcal{S}_t^R)\right] \tag{45}
$$

Define the certainty equivalent of future utility as:

$$
CE_t^R = \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^R\left(W_{t+1}^R, \mathcal{S}_{t+1}^R\right)\right)^{1-\sigma_R}\right]^{\frac{1}{1-\sigma_R}}. \tag{46}
$$

and the composite within-period utility (evaluated at $\rho = 0$) as:

$$
u_t^R = (C_t^R)^{1-\theta}(A_K K_{t-1}^R)^\theta.
$$

A.3.2 First-order conditions

Optimal Default Decision The optimization consists of choosing a function $D(\rho) : \mathbb{R} \to \{0,1\}$ that specifies for each possible realization of the penalty ρ whether or not to default.

Since the value function $V^R(W,\rho,\mathcal{S}_t^R)$ defined in (38) is increasing in wealth W and decreasing in the penalty ρ, there will generally exist an optimal threshold penalty ρ^* such that for a given W_t^R, risk-takers optimally default for all realizations $\rho < \rho^*$. Hence we can equivalently write the optimization problem in (45) as

$$
\begin{aligned}
\tilde{V}^R(W_t^R, \mathcal{S}_t^R) =& \max_{\rho^*} \mathrm{E}_\rho\left[\mathbb{1}[\rho<\rho^*]\, V^R(0,\rho,\mathcal{S}_t^R) + (1-\mathbb{1}[\rho<\rho^*])\, V^R(W_t^R,0,\mathcal{S}_t^R)\right]\\
=& \max_{\rho^*} F_\rho(\rho^*)\, \mathrm{E}_\rho\left[V^R(0,\rho,\mathcal{S}_t^R)\,|\,\rho<\rho^*\right] + (1-F_\rho(\rho^*))\, V^R(W_t^R,0,\mathcal{S}_t^R).
\end{aligned}
$$

The solution ρ_t^* is characterized by the first-order condition:

$$
V^R(0,\rho_t^*,\mathcal{S}_t^R) = V^R(W_t^S,0,\mathcal{S}_t^R).
$$

By defining the partial inverse $\mathcal{F} : (0,\infty) \to (-\infty,\infty)$ of $V^S(\cdot)$ in its second argument as

$$
\left\{(x,y) : y = \mathcal{F}(x) \Leftrightarrow x = V^R(0,y)\right\},
$$

we get that

$$
\rho_t^* = \mathcal{F}(V^R(W_t^R,0,\mathcal{S}_t^R)), \tag{47}
$$

and by substituting the solution into (45), we obtain

$$
\tilde{V}^R(W_t^R,\mathcal{S}_t^R) = F_\rho(\rho_t^*)\mathrm{E}_\rho\left[V^R(0,\rho,\mathcal{S}_t^R)\,|\,\rho<\rho_t^*\right] + (1-F_\rho(\rho_t^*))\, V^R(W_t^R,0,\mathcal{S}_t^R). \tag{48}
$$

Equations (38), (47), and (48) completely characterize the optimization problem of risk-takers.

To compute the optimal bankruptcy threshold ρ_t^*, note that the inverse value function defined in equation (47) is given by:

$$\mathcal{F}(x) = \begin{cases} \log((1-\beta_R)u_t^R) - \frac{1}{1-1/\nu}\log\left(x^{1-1/\nu} - \beta_R(CE_t^R)^{1-1/\nu}\right) & \text{for } \nu > 1 \\ (1-\beta_R)\log(u_t^R) + \beta_R\log(CE_t^R) - \log(x) - (1-\beta_R) & \text{if } \nu = 1. \end{cases}$$

Optimal Portfolio Choice The first-order condition for the short-term bond position is:

$$\begin{aligned} & q_t^f \frac{1-\theta}{C_t^R}(1-\beta_R)(V_t^R)^{1/\nu}(u_t^R)^{1-1/\nu} = \\ & \lambda_t^R + \beta_R \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^R)^{-\sigma_R}\tilde{V}_{W,t+1}^R](CE_t^R)^{\sigma_R - 1/\nu}(V_t^R)^{1/\nu} \end{aligned} \tag{49}$$

where λ_t^R is the Lagrange multiplier on the borrowing constraint (41).

The first order condition for the government-guaranteed mortgage bond position is:

$$\begin{aligned} & (q_t^m + \gamma_t)\frac{1-\theta}{C_t^R}(1-\beta_R)(V_t^R)^{1/\nu}(u_t^R)^{1-1/\nu} = \\ & \lambda_t^R \xi_G q_t^m + \mu_{G,t}^R + \beta_R \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^R)^{-\sigma_R}\tilde{V}_{W,t+1}^R\left(M_{G,t+1} + \delta Z_A(\omega_{t+1}^*)q_{t+1}^m\right)](CE_t^R)^{\sigma_R - 1/\nu}(V_t^R)^{1/\nu}, \end{aligned} \tag{50}$$

where $\mu_{t,G}^R$ is the Lagrange multiplier on the no-shorting constraint for guaranteed loans (42).

The first order condition for the private mortgage bond position is:

$$\begin{aligned} & q_t^m \frac{1-\theta}{C_t^R}(1-\beta_R)(V_t^R)^{1/\nu}(u_t^R)^{1-1/\nu} = \\ & \lambda_t^R \xi_P q_t^m + \mu_{P,t}^R + \beta_R \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^R)^{-\sigma_R}\tilde{V}_{W,t+1}^R\left(M_{P,t+1} + \delta Z_A(\omega_{t+1}^*)q_{t+1}^m\right)](CE_t^R)^{\sigma_R - 1/\nu}(V_t^R)^{1/\nu}, \end{aligned} \tag{51}$$

where $\mu_{t,P}^R$ is the Lagrange multiplier on the no-shorting constraint for guaranteed loans (43).

A.3.3 Marginal value of wealth and SDF

Differentiating (48) gives the marginal value of wealth

$$\tilde{V}_{W,t}^R = (1 - F_\rho(\rho_t^*))\,\frac{\partial V^R(W_t^R, 0, \mathcal{S}_t^R)}{\partial W_t^R},$$

where

$$\frac{\partial V^R(W_t^R, 0, \mathcal{S}_t^R)}{\partial W_t^R} = \frac{1-\theta}{C_t^R}(1-\beta_R)(V^R(W_t^R, 0, \mathcal{S}_t^R))^{1/\nu}(u_t^R)^{1-1/\nu},$$

The stochastic discount factor of risk-takers is therefore

$$\mathcal{M}_{t,t+1}^R = \beta_R e^{-\sigma_R g_{t+1}}\left(\frac{V^R(W_{t+1}^R, 0, \mathcal{S}_{t+1}^R)}{CE_t^R}\right)^{-(\sigma_R - 1/\nu)}\left(\frac{C_{t+1}^R}{C_t^R}\right)^{-1}\left(\frac{u_{t+1}^R}{u_t^R}\right)^{1-1/\nu},$$

and

$$\tilde{\mathcal{M}}_{t,t+1}^R = (1 - F_\rho(\rho_{t+1}^*))\mathcal{M}_{t,t+1}^R$$

A.3.4 Euler Equations

It is then possible to show that the FOC with respect to B_t^R, $A_{t+1,G}^R$, and $A_{t+1,P}^R$ respectively, are:

$$q_t = \tilde{\lambda}_t^R + \mathrm{E}_t\left[\tilde{\mathcal{M}}_{t,t+1}^R\right], \tag{52}$$

$$q_t^m(1 - \xi_G\tilde{\lambda}_t^R) + \gamma_t = \tilde{\mu}_{t,G} + \mathrm{E}_t\left[\tilde{\mathcal{M}}_{t,t+1}^R\left(M_{G,t+1} + \delta Z_A(\omega_{t+1}^*)q_{t+1}^m\right)\right], \tag{53}$$

$$q_t^m(1 - \xi_P\tilde{\lambda}_t^R) = \tilde{\mu}_{t,P} + \mathrm{E}_t\left[\tilde{\mathcal{M}}_{t,t+1}^R\left(M_{P,t+1} + \delta Z_A(\omega_{t+1}^*)q_{t+1}^m\right)\right]. \tag{54}$$

A.4 Equilibrium

The optimality conditions describing the problem are (16), (26) and (27) for borrowers, (28), (36), and (37) for depositors, and (39), (52), (53), and (54) for risk takers. We add complementary slackness conditions for the constraints (18) for borrowers, (30) and (31) for depositors, and (41), (42), and (43) for risk-takers. Together with the market clearing conditions, these equations fully characterize the economy.

B Calibration Appendix

B.1 States and transition Probabilities

After discretizing the aggregate real per capita income growth process as a Markov chain using the Rouwenhorst method, we obtain the following five states for g:

$$[0.943,\ 0.980,\ 1.018,\ 1.058,\ 1.101]$$

with 5×5 transition probability matrix:

$$\begin{bmatrix} 0.254 & 0.415 & 0.254 & 0.069 & 0.007 \\ 0.103 & 0.381 & 0.363 & 0.134 & 0.017 \\ 0.042 & 0.242 & 0.430 & 0.242 & 0.042 \\ 0.017 & 0.134 & 0.363 & 0.381 & 0.103 \\ 0.007 & 0.069 & 0.254 & 0.415 & 0.254. \end{bmatrix}$$

We discretize the process for σ_ω^2 into a two-state Markov chain that is correlated with income growth g. The two states are:

$$[.078, .203]$$

The transition probability matrix, conditional on being in one of the bottom two g states is:

$$\begin{bmatrix} 0.80 & 0.20 \\ 0.01 & 0.99 \end{bmatrix}$$

The transition probability matrix, conditional on being in one of the top three g states is:

$$\begin{bmatrix} 1.0 & 0.0 \\ 1.0 & 0.0 \end{bmatrix}$$

The stationary distribution for the joint Markov chain of g and σ_ω^2 is

State	1	2	3	4	5	6	7
g	0.943	0.943	0.980	0.980	1.018	1.058	1.101
σ_ω^2	0.078	0.203	0.078	0.203	0.078	0.078	0.078
Prob.	0.039	0.023	0.167	0.081	0.372	0.255	0.063

From a long simulation, we obtain the following mean, standard deviation, and persistence for g: 1.019, .039, and .42, respectively. We obtain the following mean, standard deviation, and persistence for σ_ω^2: .092, .039, and .46, respectively. We obtain a correlation between g and σ_ω of -0.42.

B.2 Evidence on default rates and mortgage severities

Since not all mortgage delinquencies result in foreclosures (loans can cure or get modified), we use the fraction of loans that 90-day or more delinquent or in foreclosure as the real world counterpart to our model's default rate. Some loans that were 90-day delinquent or more received a loan modification, but many of these modifications resulted in a redefault 12 to24 months later. Given that our model abstracts from modifications, using a somewhat broader criterion of delinquency than foreclosures-only seems warranted.

The observed 90-day plus (including foreclosures) default rate rose from 2% at the start of 2007 to just under 10% in 2010.Q1. Since then, the default rate has been gradually falling back, to 4.7% by 2014.Q3 (Mortgage Bankers Association and Urban Institute). The slow decline in foreclosure rates in the data is partly due to legal delays in the foreclosure process, especially in judicial states like New York and Florida where the average foreclosure process takes up to 1000 days. In other part it is due to re-defaults on modified loans. Since, neither is a feature of the model, it seems reasonable to interpret the abnormally high default rates of the post-2013 period as due to such delays, and to reassign them to the 2010-2012 period. If we assume that the foreclosure rate will return to its normal 2% level by the end of 2016, then such reassignment delivers an average foreclosure rate of 8.5% during the 2007-2012 foreclosure crisis. Absent reassignment, the average default rate would be 5.9% over the 2007-2016 period. Jeske et al. (2014) target only a 0.5% foreclosure rate, but their calibration is to the pre-2006 sample. The evidence from the post-2006 period dramatically raises the long-term mean default rate.

Fannie Mae's 10K filings for 2007 to 2013 show that severities, or losses-given-default, on conventional single-family loans were 4% in 2006, 11% in 2007, 26% in 2008, 37% in 2009, 34% in 2010, 35% in 2011, 31% in 2012, and 24% in 2013. Severities on Fannie's non-conforming (mostly Alt-A and subprime) portfolio holdings exceed 60% in all these years. If anything, the severity rate on Fannie's *non-conforming* holdings is lower than that of the overall non-conforming market due to advantageous selection (Adelino et al 2014). Given that the non-conforming market accounted for half of all mortgage originations in 2004-2007, the severities on conventional loans are too low to accurately reflect the market-wide severities. To take account of this composition effect, we target a market-wide severity rate of 40% in the crisis (2007-2012). We target a severity rate of 15% in non-crisis years (pre-2007 and post-2012), based on Fannie's experience in that period and the much smaller size of the non-conforming mortgage market in those years.

Combining a default rate of 2% in normal times with a severity of 15%, we obtain a loss rate of 0.3% in normal times. Combining the default rate of 8.5% during a foreclosure crisis with the severity of 40% in crises, we obtain a 3.4% loss rate.

To obtain mortgage debt to GDP in normal times and in crisis times, we calculate a time series of household mortgage debt (including debt on mutli-family real estate owned by the household sector) and divide by GDP. Since mortgage debt-GDP saw a gradual decrease for reasons related to new technology, such as automated underwriting and securitization, we focus attention on the post-1985 period. Mortgage debt-GDP averages to 54% in the 1985-1999 period. We target this for our normal times value. Mortgage debt-GDP averages to 78% in the 2000-2014 period. We target that number for our crisis number.

B.3 Long-term mortgages

Our model's mortgages are geometrically declining perpetuities, and as such have no principal. The issuer of one unit of the bond at time t promises to pay the holder 1 at time $t+1$, δ at time $t+2$, δ^2 at time $t+3$, and so on. If the borrower defaults on the mortgage, the government guarantee entitles the holder to receive a "principal repayment" $F = \frac{\alpha}{1-\delta}$, a constant parameter that does not depend on the value of the collateral or any state variable of the economy. Real life mortgages have a finite maturity (usually 30 years) and a principal payment. They also have a vintage (year of origination), whereas our mortgages combine all vintages in one variable. This appendix explains how to map the geometric mortgages in our model into real-world mortgages.

Our model's mortgage refers to the entire pool of all outstanding mortgages. In reality, this pool not only consists of newly issued 30-year fixed-rate mortgages (FRMs), but also of newly issued 15-year mortgages, other mortgage types such as hybrid adjustable-rate mortgages (ARMs), as well as all prior vintages of all mortgage types. This includes, for example, 30 year FRMs issued 29 years ago. The Barclays U.S. Mortgage Backed Securities (MBS) Index is the best available measure of the overall pool of outstanding government-guaranteed mortgages. It tracks agency mortgage backed pass-through securities (both fixed-rate and hybrid ARM) guaranteed by Ginnie Mae (GNMA), Fannie Mae (FNMA), and Freddie Mac (FHLMC). The index is constructed by grouping individual TBA-deliverable MBS pools into aggregates or generics based on program, coupon and vintage. For this MBS index we obtain a time series of monthly price, duration (the sensitivity of prices to interest rates), weighted-average life (WAL), and weighted-average coupon (WAC) for January 1989 until December 2014.

Our calibration strategy is to choose values for δ and F so that the relationship between price and interest rate (duration) is the same for the observed Barclays MBS Index and for the model's geometric bond. We proceed in two steps. In the first step, we construct a simple model to price a pool of MBS bonds and calibrate it to match the observed time series of MBS durations. With this auxiliary model in hand, we then choose the two parameters to match the price-rate curve in the auxiliary model and the geometric mortgage model.

B.3.1 Step 1: A simple MBS pricing model

Changes in duration of the Barclays MBS index are often driven by changes in the index composition. As mortgages are prepaid and new ones are issued with different coupons, both the weighted-average-life and weighted-average-coupon of the Index change significantly. Any model that wants to have a chance at matching the observed durations must account of these compositional changes.

For simplicity, we assume that all mortgages are 30-year fixed-rate mortgages. We construct a portfolio of MBS with *remaining* maturities ranging from 1 to 360 months. Each month, a fraction of each MBS prepays. We assume that the prepayment rate is given by a function $CPR(c-r)$ which depends on the "prepayment incentive" of that particular MBS, defined as the difference between the original coupon rate of that mortgage and the current mortgage rate. We assume that every prepayment is a refinancing: a dollar of mortgage balance prepaid result in a dollar of new mortgage balance originated at the new mortgage rate. In addition, each period an exogenously given amount of new mortgages are originated with a coupon equal to that month's mortgage rate to reflect purchase originations (as opposed to refinancing originations).

In a given month t, each mortgage i has starting balance bal_t^i, pays a monthly mortgage pmt_t^i of which int_t^i is interest and $prin_t^i$ is scheduled principal, where i is the remaining maturity of the mortgage, i.e., the mortgage was originated at time $t-(360-i)-1$. Denote the unscheduled principal payments, or prepayments, by prp_t^i. Let SMM_t^i be the prepayment rate in month t on that mortgage. The evolution equations for actual mortgage cash flows are:

$$\begin{aligned}
int_t^i &= \frac{c_{t-(360-i)-1}}{12} \times bal_t^i \\
prin_t^i &= pmt_t^i - int_t^i \\
prp_t^i &= SMM_t^i(bal_t^i - prin_t^i) \\
bal_{t+1}^{i-1} &= (1 - SMM_t^i)(bal_t^i - prin_t^i) \\
pmt_{t+1}^{i-1} &= (1 - SMM_t^i)pmt_t^i
\end{aligned}$$

The initial payment is given by the standard annuity formula, normalizing the amount borrowed to 1.

$$\begin{aligned}
pmt_t^{360} &= \frac{\frac{c_{t-1}}{12}}{1 - (1 + c_{t-1}/12)^{-360}} \\
bal_t^{360} &= 1 + \sum_{i=1}^{360} prp_{t-1}^i
\end{aligned}$$

The last equation says that the initial balance of new 30-year FRMs is comprised on 1 unit of purchase originations, an exogenously given flow of originations each period, plus refinancing originations which equal all prepayments from the previous period.

Furthermore, at every month t we compute *projected* cash flows on each mortgage assuming mortgage rates stay constant from t until maturity i. These projected cash flows follow the same evolution equations as presented above. Denote these projected cash flows with a tilde over the variable.

We can then compute the price P_t, (modified) duration Dur_t, and weighted-average-life WAL_t of the MBS *portfolio* comprised of all vintages:

$$\begin{aligned}
P_t &= \sum_{i=1}^{360} \sum_{s=0}^{i} \frac{\tilde{pmt}_{t+s}^{i-s} + \tilde{prp}_{t+s}^{i-s}}{(1 + r_t/12)^s} \\
Dur_t &= \frac{1}{1 + \frac{r_t}{12}} \sum_{i=1}^{360} \frac{1}{P_t^i} \sum_{s=0}^{i} \frac{\tilde{pmt}_{t+s}^{i-s} + \tilde{prp}_{t+s}^{i-s}}{(1 + r_t/12)^s} s \\
WAL_t^i &= \sum_{i=1}^{360} \frac{\sum_{s=0}^{i}(\tilde{pmt}_{t+s}^{i-s} + \tilde{prp}_{t+s}^{i-s})s}{\sum_{i=1}^{360} \sum_{s=0}^{i}(\tilde{pmt}_{t+s}^{i-s} + \tilde{prp}_{t+s}^{i-s})}
\end{aligned}$$

What remains to be specified is our prepayment model delivering the single-month mortality SMM_t^i used above. Following practice, we assume an annual constant prepayment rate (CPR) which is a S-shaped function of the rate incentive: $CPR_t^i = CPR(r_t - c_{t-(360-i)-1})$:

$$CPR(x) = \underline{CPR} + (\overline{CPR} - \underline{CPR}) \left(1 - \frac{\exp(\psi(x - \bar{x})}{1 + \exp(\psi(x - \bar{x}))}\right)$$

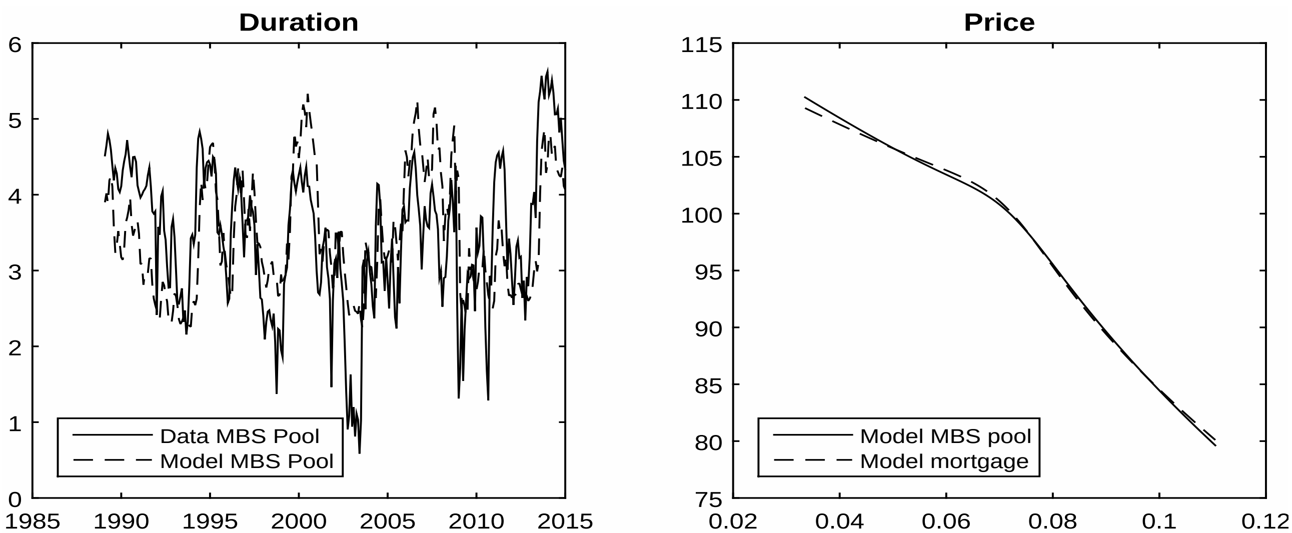
The annual CPR implies a monthly SMM $SMM_t^i = factor_i \times (1 - (1 - (CPR_t^i)^{1/12})$. The multiplicative $factor_i$ allows us to deal with slow prepayments early in the life of the mortgage (the "ramp-up" phase) and late in the life of the mortgage (the "burn-out" phase). For simplicity, we make $factor_i$ linearly increasing from 0 in month 1 (when $i = 360$) to 1 in month 30, flat at 1 between month 30 and month 180 and linearly decreasing back to 0 between months 180 and month 360. We choose the CPR curve parameters $\{\underline{CPR}, \overline{CPR}, \psi, \bar{x}\}$ to minimize the sum of squared errors between the time series of model-implied duration $\{Dur_t\}$ and observed duration on the Barclays index.

To produce the time-series of model-implied duration $\{Dur_t\}$, we feed in the observed 30-year conventional fixed rate mortgage rate (MORTGAGE30US in FRED), $\{r_t\}$. We initialize the portfolio many years before the start of our time series data to ensure that the model is in steady state by the time our time series for the Barclays index starts. Specifically, we start the computation in April 1903 by issuing 1 MBS. By March 1933, we have a complete portfolio of 360 fixed-rate amortizing mortgages, maturing any month from April 1933 to March 1963.

The left panel of Figure 2 shows the observed time series of duration on the Barclays MBS index plotted against the model-implied duration on the MBS pool. The two time series track each other quite closely despite several strong modeling assumptions. The resulting CPR curve looks close to historical average prepayment behavior on agency MBS, as prepayment data from SIFMA indicate. CPR is slightly above 40% when the rate incentive is 200 basis points or more, about 15% when the rate incentive is zero, and slightly above 5% when the rate incentive is below -200 basis points.

Figure 2: Matching Mortgages in Model to Data

The left panel plots the observed time series of duration on the Barclays MBS index (solid line) plotted against the duration on the model-implied MBS pool (dashed-line). The right panel plots the mortgage price-interest rate relationship for the model-implied MBS pool (solid line) and the model-implied geometrically declining perpetual mortgage (dashed line). Prices on a $100 face value mortgage are on the vertical axis, while interest rates are on the horizontal axis. The Barclays MBS index data are from Bloomberg for the period 1989 until 2014 (daily frequency). The calculations also use the 30-year fixed-rate mortgage rate from FRED.



B.3.2 Step 2: Matching MBS pool to perpetual mortgage in our model

With a well-calibrated auxiliary model for a MBS pool, we now proceed to match key features of that auxiliary model's MBS pool to the mortgage in our model, which is a geometrically declining perpetuity.

We start by computing the price $P(r)$ of a fixed-rate MBS with maturity T and coupon c as a function of the current mortgage rate r, using the constant prepayment rate function $C\hat{P}R(r) = CPR(r - c)$ obtained from step 1. For T and c we use the time-series average of the weighted-average maturity and weighted-average coupon, respectively, from the model-implied MBS pool obtained in step 1.

We can write the steady-state price of a geometric mortgage with parameters (δ, F) recursively as:

$$Q(r) = \frac{1}{1+r}\Big(1 - C\hat{P}R(r) + C\hat{P}R(r)F + (1 - C\hat{P}R(r))\delta Q(r)\Big)$$

Solving for $Q(r)$, we get

$$Q(r) = \frac{1 - C\hat{P}R(r) + C\hat{P}R(r)F}{1 + r - \delta(1 - C\hat{P}R(r))}$$

The stage 2 calibration determines how many units X of the geometric mortgage with what parameters (δ, F) one needs to sell to hedge one unit of the MBS against parallel shifts in interest rates, across the range of historical mortgage rates:

$$\min_{\delta,F,X} \int [P(r) - XQ(r)]^2 dr$$

We estimate values of $\delta = 0.967$, $F = 10.675$, which implies $\alpha = 0.3524$, and $X = 0.1022$. For the model calibration, we only need δ and α. The right panel of Figure 2 shows that the fit is excellent. The average error is only 0.28% of the MBS pool price.

In conclusion, despite its simplicity, the perpetual mortgage in the model captures all important features of real life mortgages (or MBS pools). The relationship between price and interest rate is convex when rates are high and concave ("negative convexity") when rates are low, which is when the prepayment option is in the money. It matches the interest rate risk (duration) of real-life mortgages, for different interest rate scenarios.