International Capital Flows: Private versus Public*

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Abstract

We study both empirically and quantitatively the patterns of international capital flows by the private sector and the public sector.

JEL Classifications: F11, F43, O33, O47

Keywords: default risk, private capital flows, public capital flows

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1 Introduction

2 Data

2.1 Growth and Capital Flows (OLD)

The figures show the relationships between real per-capita GDP growth and capital inflows in developing countries between 1980 and 2010. Total capital inflow is measured as the average of $-CA_t/GDP_t$, public capital inflow is the average of $(\Delta PPG_t - \Delta Reserves_t)/GDP_t$, private capital inflow is total capital inflow minus public capital inflow, and reserve accumulation is the average of $\Delta Reserves_t/GDP_t$. The sample includes 116 developing countries.

There is no significant relationship between growth and total capital inflows. However, once we divide total capital inflows into private and public, one can see a strong positive correlation between growth and private inflows, while there is a slightly negative correlation between growth and public inflows. Public outflows in countries with high growth seem to have been driven by reserve accumulation as shown in subfigure d. Private capital actually flowed into developing countries with high growth as predicted by the neoclassical growth model. We can see similar patterns across country groups with different exchange rate regimes.
Figure 1: Growth and Capital Inflows (All Developing Countries: 1980-2010)

(a) Growth and Total Capital Inflow
(b) Growth and Private Capital Inflow
(c) Growth and Public Capital Inflow
(d) Growth and Reserve Accumulation

Table 1: Correlation between Per-capita GDP Growth and Measures of Capital Inflows

<table>
<thead>
<tr>
<th></th>
<th>Total Capital Inflow</th>
<th>Private Inflow</th>
<th>Public Inflow</th>
<th>Reserve Accumulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Developing Countries</td>
<td>0.125 (0.184)</td>
<td>0.545*** (0.157)</td>
<td>-0.150* (0.0814)</td>
<td>0.677*** (0.126)</td>
</tr>
<tr>
<td>Countries with Fixed Exchange Regime</td>
<td>0.268 (0.229)</td>
<td>0.532*** (0.191)</td>
<td>-0.039 (0.0893)</td>
<td>0.711*** (0.166)</td>
</tr>
<tr>
<td>Countries with Floating Exchange Regime</td>
<td>-0.259 (0.286)</td>
<td>0.573** (0.280)</td>
<td>-0.430** (0.173)</td>
<td>0.609*** (0.138)</td>
</tr>
</tbody>
</table>

Note: Standard errors are in parentheses.
*: p-value less than 10%, **: p-value less than 5%, ***: p-value less than 1%
Comparison with Gourinchas and Jeanne’s (2012) allocation puzzle paper: Their data coverage is from 1980 to 2000, while ours is from 1980 to 2010. Their x-axis is productivity growth, while ours is per-capita GDP growth. Most of all, they include 67 countries based on the Penn World Tables version 6.1 (denote these counties by the PWT sample), while ours include more than 100 countries. Alfaro, Kalemli-Ozcan and Volosovych (2011) document correlations between per-capita GDP growth and capital flows for 1980-2007. They also have (insignificant) positive correlation between growth and total capital inflows as ours. They point out that if they only include countries in the PWT sample, they get strongly negative correlation between growth and total capital inflow as in Gourinchas and Jeanne (2012). See Table 1 and Figure 6 of Alfaro et al. (2011).

!!!

NOTE: here we need some further diagnosis. (1) Keep the same sample as GJ, but use output growth instead of TFP growth. This is to illustrate that GJ results might still hold with GDP growth instead TFP frowth. (2) Use the same country sample as GJ, vary the time sample to see whether the new decade that we are using changes the results substantially. (3) Use the same time period sample as GJ, vary the country sample to see whether the country sample changes their results.
Figure 2: Growth and Capital Inflows (Countries with Fixed Exchange Regime: 1980-2010)

(a) Growth and Total Capital Inflow
(b) Growth and Private Capital Inflow
(c) Growth and Public Capital Inflow
(d) Growth and Reserve Accumulation
2.2 Data Description

(Our measure are the same as those used in Alfaro, Kalemli-Ozcan and Volosovych (2014))

- Since Current Account = (Flows of Private Assets - Flows of Private Liabilities) + (Flows of Public Assets - Flows of Public Liabilities), we define net (total) capital inflow as follows:

  Net (total) capital inflow: an average of the annual observations for the negative of the current account balance from the IFS normalized by the annual nominal GDP from WDI, both in U.S. dollars. That is, \( \text{Avg}(-CA_t/GDP_t) \).

- Net private flows are calculated by the “residual method,” as the difference of the total net capital flows and net public debt flows. Thus,

  Net private flows = Net total flows - Net public debt flows
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- Net public debt flows are net PPG debt flows - reserve accumulation: \((PPG_t - PPG_{t-1}) - (\text{Reserves}_t - \text{Reserves}_{t-1})\),

  where Reserves is reserves minus gold from WDI

- All debt flows are normalized by \(GDP_t\) to be consistent with total net flows.

- Growth: The average of annual growth of the real GDP per capita from WDI. \(\text{Avg}\left(\frac{Y_t - Y_{t-1}}{Y_t}\right)\).

  Note that here we normalize by \(GDP_t\) too.

- Note, Gourinchas and Jeanne’s (2013) measure of capital inflow: \(\frac{\Delta D}{Y_0} = \frac{D_{2000} - D_{1980}}{Y_{1980}}\) where \(D_{2000} - D_{1980}\) is cumulative current account deficits.

- Our sample includes developing countries (classified by the world bank) whose debt data are available in the GDF data set. The sample period is 1980-2010, but we include countries if they start reporting relevant variables before 1990. The sample includes 116 developing countries.

Jing’s recap of the empirical results: For countries with higher output growth rates, the private sector borrows more from abroad, while the public sector borrows less (or saves more in reserves). Similarly, for countries with lower output growth rates, the private sector borrows less from abroad, while the public sector borrows more (or dissaves more in reserves). These patterns lead to a weak correlation between output growth and total capital inflows. They also are robust to the countries with different exchange rate regimes.
2.3 Public v.s. Private Capital Flows (NEW)

Now positive flow means inflow, negative flow means outflow.

2.3.1 All Sample Developing Countries

![Figure 4: Public and Private Capital Flows](image)

- (a) All Developing Countries
- (b) Countries with Public Borrowing
- (c) Countries with Public Saving

![Figure 5: Public and Private Capital Flows for Varying Public Saving Levels](image)

- (a) High Public Borrowing
- (b) Low Public Borrowing
- (c) Low Public Saving
- (d) High Public Saving

### Table 2: Regression of Private Flow on Public Flow

<table>
<thead>
<tr>
<th></th>
<th>All (1)</th>
<th>$A_2 &lt; 0$ (2)</th>
<th>$A_2 &gt; 0$ (3)</th>
<th>high borrowing (4)</th>
<th>low borrowing (5)</th>
<th>low saving (6)</th>
<th>high saving (7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PublicFlow</td>
<td>-0.783***</td>
<td>-0.0578</td>
<td>-3.443***</td>
<td>-0.546</td>
<td>0.475</td>
<td>-2.760</td>
<td>-3.950*</td>
</tr>
<tr>
<td></td>
<td>(0.226)</td>
<td>(0.257)</td>
<td>(0.639)</td>
<td>(0.382)</td>
<td>(1.208)</td>
<td>(2.278)</td>
<td>(1.615)</td>
</tr>
<tr>
<td>Observations</td>
<td>121</td>
<td>99</td>
<td>22</td>
<td>34</td>
<td>65</td>
<td>15</td>
<td>7</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.091</td>
<td>0.001</td>
<td>0.592</td>
<td>0.060</td>
<td>0.002</td>
<td>0.101</td>
<td>0.545</td>
</tr>
</tbody>
</table>
Figure 6: Public and Private Capital Flows (Outlier removed)

(a) All Developing Countries (Out removed)
(b) Countries with Public Borrowing (Out removed)
(c) Countries with Public Saving

Figure 7: Public and Private Capital Flows for Varying Public Saving Levels (Outlier removed)

(a) High Public Borrowing (b) Low Public Borrowing (c) Low Public Saving (d) High Public Saving

Table 3: Regression of Private Flow on Public Flow

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>$A_2 &lt; 0$</th>
<th>$A_2 &gt; 0$</th>
<th>High Borrowing</th>
<th>Low Borrowing</th>
<th>Low Saving</th>
<th>High Saving</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PublicFlow</strong></td>
<td>-0.839***</td>
<td>1.117***</td>
<td>-3.443***</td>
<td>0.883</td>
<td>-1.973</td>
<td>-2.760</td>
<td>-3.950*</td>
</tr>
<tr>
<td></td>
<td>(0.302)</td>
<td>(0.413)</td>
<td>(0.639)</td>
<td>(0.838)</td>
<td>(1.257)</td>
<td>(2.278)</td>
<td>(1.615)</td>
</tr>
<tr>
<td><strong>Observations</strong></td>
<td>119</td>
<td>97</td>
<td>22</td>
<td>41</td>
<td>56</td>
<td>15</td>
<td>7</td>
</tr>
<tr>
<td><strong>R-squared</strong></td>
<td>0.062</td>
<td>0.071</td>
<td>0.592</td>
<td>0.028</td>
<td>0.044</td>
<td>0.101</td>
<td>0.545</td>
</tr>
</tbody>
</table>
2.3.2 Asia, Latin America, Europe (Africa Excluded)

Figure 8: Public and Private Capital Flows

(a) All Developing Countries  (b) Countries with Public Borrowing  (c) Countries with Public Saving

Figure 9: Public and Private Capital Flows for Varying Public Saving Levels

(a) High Public Borrowing  (b) Low Public Borrowing  (c) Low Public Saving  (d) High Public Saving

Table 4: Regression of Private Flow on Public Flow

<table>
<thead>
<tr>
<th></th>
<th>All (1)</th>
<th>$A_2 &lt; 0$ (2)</th>
<th>$A_2 &gt; 0$ (3)</th>
<th>high borrowing (4)</th>
<th>low borrowing (5)</th>
<th>low saving (6)</th>
<th>high saving (7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PublicFlow</td>
<td>-0.351</td>
<td>-0.0161</td>
<td>-3.362***</td>
<td>-0.979</td>
<td>0.758</td>
<td>5.501</td>
<td>-2.789*</td>
</tr>
<tr>
<td></td>
<td>(0.324)</td>
<td>(0.413)</td>
<td>(0.706)</td>
<td>(0.628)</td>
<td>(1.676)</td>
<td>(3.748)</td>
<td>(1.152)</td>
</tr>
<tr>
<td>Observations</td>
<td>67</td>
<td>53</td>
<td>14</td>
<td>18</td>
<td>35</td>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.018</td>
<td>0.000</td>
<td>0.654</td>
<td>0.132</td>
<td>0.006</td>
<td>0.235</td>
<td>0.661</td>
</tr>
</tbody>
</table>
Figure 10: Public and Private Capital Flows (Outlier removed)

(a) All Developing Countries  (b) Countries with Public Borrowing  (c) Countries with Public Saving

Figure 11: Public and Private Capital Flows for Varying Public Saving Levels (Outlier removed)

(a) High Public Borrowing  (b) Low Public Borrowing  (c) Low Public Saving  (d) High Public Saving

Table 5: Regression of Private Flow on Public Flow

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>$A_2 &lt; 0$</th>
<th>$A_2 &gt; 0$</th>
<th>high borrowing</th>
<th>low borrowing</th>
<th>low saving</th>
<th>high saving</th>
</tr>
</thead>
<tbody>
<tr>
<td>PublicFlow</td>
<td>0.239</td>
<td>2.188***</td>
<td>-3.362***</td>
<td>2.491</td>
<td>-2.318</td>
<td>5.501</td>
<td>-2.789*</td>
</tr>
<tr>
<td></td>
<td>(0.459)</td>
<td>(0.679)</td>
<td>(0.706)</td>
<td>(1.899)</td>
<td>(1.437)</td>
<td>(3.748)</td>
<td>(1.152)</td>
</tr>
<tr>
<td>Observations</td>
<td>66</td>
<td>52</td>
<td>14</td>
<td>23</td>
<td>29</td>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.004</td>
<td>0.172</td>
<td>0.654</td>
<td>0.076</td>
<td>0.088</td>
<td>0.235</td>
<td>0.661</td>
</tr>
</tbody>
</table>
2.4 Non-Official Public v.s. Private Capital Flows (NEW)

Now positive flow means inflow, negative flow means outflow.

2.4.1 All Sample Developing Countries

![Figure 12: Public and Private Capital Flows](image)

(a) All Developing Countries  (b) Countries with Public Borrowing  (c) Countries with Public Saving

![Figure 13: Public and Private Capital Flows for Varying Public Saving Levels](image)

(a) High Public Borrowing  (b) Low Public Borrowing  (c) Low Public Saving  (d) High Public Saving

<table>
<thead>
<tr>
<th>Table 6: Regression of Private Flow on Public Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>PublicFlow</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Observations</td>
</tr>
<tr>
<td>R-squared</td>
</tr>
</tbody>
</table>
Figure 14: Public and Private Capital Flows (Outlier removed)

(a) All Developing Countries (Out removed)  
(b) Countries with Public Borrowing  (Out removed)  
(c) Countries with Public Saving

Figure 15: Public and Private Capital Flows for Varying Public Saving Levels (Outlier removed)

(a) High Public Borrowing  
(b) Low Public Borrowing  
(c) Low Public Saving  
(d) High Public Saving

Table 7: Regression of Private Flow on Public Flow

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>$A_2 &lt; 0$</th>
<th>$A_2 &gt; 0$</th>
<th>high borrowing</th>
<th>low borrowing</th>
<th>low saving</th>
<th>high saving</th>
</tr>
</thead>
<tbody>
<tr>
<td>PublicFlow</td>
<td>-1.876***</td>
<td>0.645</td>
<td>-3.130***</td>
<td>1.797</td>
<td>10.24</td>
<td>-2.249</td>
<td>-3.336*</td>
</tr>
<tr>
<td></td>
<td>(0.605)</td>
<td>(2.746)</td>
<td>(0.762)</td>
<td>(7.631)</td>
<td>(13.02)</td>
<td>(2.045)</td>
<td>(1.783)</td>
</tr>
<tr>
<td>Observations</td>
<td>115</td>
<td>26</td>
<td>89</td>
<td>10</td>
<td>16</td>
<td>57</td>
<td>32</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.078</td>
<td>0.002</td>
<td>0.163</td>
<td>0.007</td>
<td>0.042</td>
<td>0.022</td>
<td>0.105</td>
</tr>
</tbody>
</table>
2.4.2 Asia, Latin America, Europe (Africa Excluded)

Figure 16: Public and Private Capital Flows

(a) All Developing Countries  (b) Countries with Public Borrowing  (c) Countries with Public Saving

Figure 17: Public and Private Capital Flows for Varying Public Saving Levels

(a) High Public Borrowing  (b) Low Public Borrowing  (c) Low Public Saving  (d) High Public Saving

Table 8: Regression of Private Flow on Public Flow

<table>
<thead>
<tr>
<th></th>
<th>All (1)</th>
<th>$A_2 &lt; 0$ (2)</th>
<th>$A_2 &gt; 0$ (3)</th>
<th>high borrowing (4)</th>
<th>low borrowing (5)</th>
<th>low saving (6)</th>
<th>high saving (7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PublicFlow</td>
<td>-1.155***</td>
<td>-0.850</td>
<td>-2.857***</td>
<td>-1.631</td>
<td>0.588</td>
<td>-1.970</td>
<td>-2.497</td>
</tr>
<tr>
<td></td>
<td>(0.383)</td>
<td>(0.603)</td>
<td>(0.833)</td>
<td>(1.615)</td>
<td>(3.695)</td>
<td>(2.606)</td>
<td>(1.842)</td>
</tr>
<tr>
<td>Observations</td>
<td>64</td>
<td>21</td>
<td>43</td>
<td>4</td>
<td>17</td>
<td>27</td>
<td>16</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.128</td>
<td>0.095</td>
<td>0.223</td>
<td>0.338</td>
<td>0.002</td>
<td>0.022</td>
<td>0.116</td>
</tr>
</tbody>
</table>
Figure 18: Public and Private Capital Flows (Outlier removed)

(a) All Developing Countries  
(b) Countries with Public Borrowing  
(c) Countries with Public Saving

Figure 19: Public and Private Capital Flows for Varying Public Saving Levels (Outlier removed)

(a) High Public Borrowing  
(b) Low Public Borrowing  
(c) Low Public Saving  
(d) High Public Saving

Table 9: Regression of Private Flow on Public Flow

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>(A_2 &lt; 0)</th>
<th>(A_2 &gt; 0)</th>
<th>high borrowing</th>
<th>low borrowing</th>
<th>low saving</th>
<th>high saving</th>
</tr>
</thead>
<tbody>
<tr>
<td>PublicFlow</td>
<td>-1.469**</td>
<td>4.175</td>
<td>-2.857***</td>
<td>9.769</td>
<td>5.273</td>
<td>-1.970</td>
<td>-2.497</td>
</tr>
<tr>
<td></td>
<td>(0.677)</td>
<td>(3.064)</td>
<td>(0.833)</td>
<td>(8.221)</td>
<td>(12.63)</td>
<td>(2.606)</td>
<td>(1.842)</td>
</tr>
<tr>
<td>Observations</td>
<td>63</td>
<td>20</td>
<td>43</td>
<td>9</td>
<td>11</td>
<td>27</td>
<td>16</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.072</td>
<td>0.094</td>
<td>0.223</td>
<td>0.168</td>
<td>0.019</td>
<td>0.022</td>
<td>0.116</td>
</tr>
</tbody>
</table>
### 2.5 Summary Statistics on Capital Flows

#### Table 10: Capital Flows by Public Inflows (Including public inflows from official creditors)

<table>
<thead>
<tr>
<th></th>
<th>Total Flow</th>
<th>Private Flow</th>
<th>Public Flow</th>
<th>Non-official Public Flow</th>
<th>Reserve Accumulation</th>
<th>Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>All (118 countries)</strong></td>
<td>Mean</td>
<td>4.61%</td>
<td>-5.33%</td>
<td>-1.61%</td>
<td>0.55%</td>
<td>-0.41%</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>4.59%</td>
<td>-3.45%</td>
<td>-1.46%</td>
<td>0.62%</td>
<td>0.07%</td>
</tr>
<tr>
<td><strong>Public Inflow (96 countries)</strong></td>
<td>Mean</td>
<td>6.30%</td>
<td>-4.43%</td>
<td>-2.38%</td>
<td>0.18%</td>
<td>-0.91%</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>5.06%</td>
<td>-3.19%</td>
<td>-1.66%</td>
<td>0.38%</td>
<td>-0.19%</td>
</tr>
<tr>
<td><strong>Public Outflow (22 countries)</strong></td>
<td>Mean</td>
<td>-0.16%</td>
<td>-9.41%</td>
<td>1.90%</td>
<td>2.14%</td>
<td>1.02%</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>3.17%</td>
<td>-4.72%</td>
<td>0.90%</td>
<td>1.84%</td>
<td>0.62%</td>
</tr>
<tr>
<td><strong>Large Public Inflow (33 countries)</strong></td>
<td>Mean</td>
<td>9.50%</td>
<td>-5.96%</td>
<td>-4.65%</td>
<td>-0.17%</td>
<td>-1.54%</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>9.18%</td>
<td>-5.21%</td>
<td>-3.47%</td>
<td>0.38%</td>
<td>0.38%</td>
</tr>
<tr>
<td><strong>Small Public Inflow (63 countries)</strong></td>
<td>Mean</td>
<td>4.63%</td>
<td>-3.63%</td>
<td>-1.21%</td>
<td>0.37%</td>
<td>-0.58%</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>3.81%</td>
<td>-2.57%</td>
<td>-1.23%</td>
<td>0.38%</td>
<td>-0.44%</td>
</tr>
<tr>
<td><strong>Small Public Outflow (15 countries)</strong></td>
<td>Mean</td>
<td>5.13%</td>
<td>-5.66%</td>
<td>0.64%</td>
<td>1.45%</td>
<td>0.26%</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>3.69%</td>
<td>-4.67%</td>
<td>0.52%</td>
<td>1.14%</td>
<td>0.16%</td>
</tr>
<tr>
<td><strong>Large Public Outflow (7 countries)</strong></td>
<td>Mean</td>
<td>-4.12%</td>
<td>-17.44%</td>
<td>4.58%</td>
<td>3.63%</td>
<td>1.59%</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>0.33%</td>
<td>-9.41%</td>
<td>3.24%</td>
<td>3.26%</td>
<td>1.53%</td>
</tr>
</tbody>
</table>
Table 11: Capital Flows by Non-Official Public Inflows (Excluding public inflows from official creditors)

<table>
<thead>
<tr>
<th></th>
<th>Total Flow</th>
<th>Private Flow</th>
<th>Public Flow</th>
<th>Non-official Public Flow</th>
<th>Reserve Accumulation</th>
<th>Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>Mean</td>
<td>4.61%</td>
<td>-5.33%</td>
<td>-1.61%</td>
<td>0.55%</td>
<td>-0.41%</td>
</tr>
<tr>
<td>(118 countries)</td>
<td>Median</td>
<td>4.59%</td>
<td>-3.45%</td>
<td>-1.46%</td>
<td>0.62%</td>
<td>0.07%</td>
</tr>
<tr>
<td>Non-official Public Inflow</td>
<td>Mean</td>
<td>7.34%</td>
<td>-4.15%</td>
<td>-3.34%</td>
<td>-1.21%</td>
<td>0.87%</td>
</tr>
<tr>
<td>(28 countries)</td>
<td>Median</td>
<td>5.11%</td>
<td>-1.57%</td>
<td>-2.13%</td>
<td>-0.43%</td>
<td>0.61%</td>
</tr>
<tr>
<td>Non-official Public Outflow</td>
<td>Mean</td>
<td>3.89%</td>
<td>-5.69%</td>
<td>-1.09%</td>
<td>1.09%</td>
<td>-0.74%</td>
</tr>
<tr>
<td>(90 countries)</td>
<td>Median</td>
<td>4.54%</td>
<td>-4.04%</td>
<td>-1.01%</td>
<td>0.81%</td>
<td>-0.26%</td>
</tr>
<tr>
<td>Large N-O Public Inflow</td>
<td>Mean</td>
<td>11.65%</td>
<td>-5.33%</td>
<td>-5.78%</td>
<td>-4.00%</td>
<td>3.29%</td>
</tr>
<tr>
<td>(6 countries)</td>
<td>Median</td>
<td>9.22%</td>
<td>-1.40%</td>
<td>-3.81%</td>
<td>-2.05%</td>
<td>1.11%</td>
</tr>
<tr>
<td>Small N-O Public Inflow</td>
<td>Mean</td>
<td>6.17%</td>
<td>-3.83%</td>
<td>-2.68%</td>
<td>-0.45%</td>
<td>0.21%</td>
</tr>
<tr>
<td>(22 countries)</td>
<td>Median</td>
<td>4.17%</td>
<td>-1.57%</td>
<td>-1.82%</td>
<td>-0.34%</td>
<td>0.45%</td>
</tr>
<tr>
<td>Small N-O Public Outflow</td>
<td>Mean</td>
<td>5.57%</td>
<td>-4.14%</td>
<td>-1.68%</td>
<td>0.54%</td>
<td>-1.78%</td>
</tr>
<tr>
<td>(57 countries)</td>
<td>Median</td>
<td>4.45%</td>
<td>-4.04%</td>
<td>-1.45%</td>
<td>0.59%</td>
<td>-0.68%</td>
</tr>
<tr>
<td>Large N-O Public Outflow</td>
<td>Mean</td>
<td>1.94%</td>
<td>-8.14%</td>
<td>-0.13%</td>
<td>2.09%</td>
<td>0.46%</td>
</tr>
<tr>
<td>(32 countries)</td>
<td>Median</td>
<td>4.61%</td>
<td>-4.01%</td>
<td>-0.53%</td>
<td>1.65%</td>
<td>0.28%</td>
</tr>
</tbody>
</table>

Table 12: Capital Flows by Per-Capita GDP Growth

<table>
<thead>
<tr>
<th></th>
<th>Total Flow</th>
<th>Private Flow</th>
<th>Public Flow</th>
<th>Non-official Public Flow</th>
<th>Reserve Accumulation</th>
<th>Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>Mean</td>
<td>4.61%</td>
<td>-5.33%</td>
<td>-1.61%</td>
<td>0.55%</td>
<td>-0.41%</td>
</tr>
<tr>
<td>(118 countries)</td>
<td>Median</td>
<td>4.59%</td>
<td>-3.45%</td>
<td>-1.46%</td>
<td>0.62%</td>
<td>0.07%</td>
</tr>
<tr>
<td>Negative Growth</td>
<td>Mean</td>
<td>5.42%</td>
<td>-4.43%</td>
<td>-1.44%</td>
<td>0.97%</td>
<td>-2.22%</td>
</tr>
<tr>
<td>(58 countries)</td>
<td>Median</td>
<td>5.06%</td>
<td>-3.94%</td>
<td>-1.10%</td>
<td>0.81%</td>
<td>-1.18%</td>
</tr>
<tr>
<td>Positive Growth</td>
<td>Mean</td>
<td>3.90%</td>
<td>-6.19%</td>
<td>-1.76%</td>
<td>0.14%</td>
<td>1.20%</td>
</tr>
<tr>
<td>(60 countries)</td>
<td>Median</td>
<td>4.57%</td>
<td>-3.03%</td>
<td>-1.65%</td>
<td>0.26%</td>
<td>0.96%</td>
</tr>
<tr>
<td>Large Negative Growth</td>
<td>Mean</td>
<td>7.21%</td>
<td>-5.51%</td>
<td>-1.26%</td>
<td>1.13%</td>
<td>-3.43%</td>
</tr>
<tr>
<td>(21 countries)</td>
<td>Median</td>
<td>7.49%</td>
<td>-4.67%</td>
<td>-1.10%</td>
<td>0.88%</td>
<td>-1.70%</td>
</tr>
<tr>
<td>Small Negative Growth</td>
<td>Mean</td>
<td>4.53%</td>
<td>-3.83%</td>
<td>-1.55%</td>
<td>0.88%</td>
<td>-1.61%</td>
</tr>
<tr>
<td>(37 countries)</td>
<td>Median</td>
<td>4.78%</td>
<td>-3.58%</td>
<td>-1.08%</td>
<td>0.69%</td>
<td>-0.90%</td>
</tr>
<tr>
<td>Small Positive Growth</td>
<td>Mean</td>
<td>4.73%</td>
<td>-2.84%</td>
<td>-1.86%</td>
<td>0.24%</td>
<td>0.10%</td>
</tr>
<tr>
<td>(38 countries)</td>
<td>Median</td>
<td>3.28%</td>
<td>-1.62%</td>
<td>-1.64%</td>
<td>0.23%</td>
<td>0.39%</td>
</tr>
<tr>
<td>Large Positive Growth</td>
<td>Mean</td>
<td>2.61%</td>
<td>-11.87%</td>
<td>-1.60%</td>
<td>-0.03%</td>
<td>2.89%</td>
</tr>
<tr>
<td>(22 countries)</td>
<td>Median</td>
<td>8.83%</td>
<td>-8.42%</td>
<td>-1.72%</td>
<td>0.37%</td>
<td>1.73%</td>
</tr>
</tbody>
</table>
### Table 13: Capital Flows by Private Flows

<table>
<thead>
<tr>
<th></th>
<th>Total Flow</th>
<th>Private Flow</th>
<th>Public Flow</th>
<th>Non-official Reserve Flow</th>
<th>Reserve Accumulation</th>
<th>Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>All (118 countries)</strong></td>
<td>Mean</td>
<td>4.61%</td>
<td>-5.33%</td>
<td>-1.61%</td>
<td>0.55%</td>
<td>-0.41%</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>4.59%</td>
<td>-3.45%</td>
<td>-1.46%</td>
<td>0.62%</td>
<td>0.07%</td>
</tr>
<tr>
<td><strong>Private Inflow (100 countries)</strong></td>
<td>Mean</td>
<td>7.40%</td>
<td>-6.85%</td>
<td>-1.30%</td>
<td>0.74%</td>
<td>-1.03%</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>5.61%</td>
<td>-4.72%</td>
<td>-1.38%</td>
<td>0.62%</td>
<td>-0.26%</td>
</tr>
<tr>
<td><strong>Private Outflow (18 countries)</strong></td>
<td>Mean</td>
<td>-4.27%</td>
<td>2.84%</td>
<td>-3.16%</td>
<td>-0.49%</td>
<td>1.57%</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>0.26%</td>
<td>1.49%</td>
<td>-1.62%</td>
<td>0.37%</td>
<td>0.66%</td>
</tr>
<tr>
<td><strong>Large Private Inflow (32 countries)</strong></td>
<td>Mean</td>
<td>13.56%</td>
<td>-14.81%</td>
<td>-1.42%</td>
<td>1.05%</td>
<td>-2.10%</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>11.85%</td>
<td>-11.25%</td>
<td>-1.72%</td>
<td>0.87%</td>
<td>-0.69%</td>
</tr>
<tr>
<td><strong>Small Private Inflow (68 countries)</strong></td>
<td>Mean</td>
<td>4.46%</td>
<td>-3.05%</td>
<td>-1.25%</td>
<td>0.59%</td>
<td>-0.52%</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>4.07%</td>
<td>-3.00%</td>
<td>-1.37%</td>
<td>0.54%</td>
<td>-0.12%</td>
</tr>
<tr>
<td><strong>Small Private Outflow (12 countries)</strong></td>
<td>Mean</td>
<td>2.03%</td>
<td>1.06%</td>
<td>-2.67%</td>
<td>0.56%</td>
<td>-0.26%</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>0.99%</td>
<td>1.14%</td>
<td>-1.64%</td>
<td>0.37%</td>
<td>-0.60%</td>
</tr>
<tr>
<td><strong>Large Private Outflow (6 countries)</strong></td>
<td>Mean</td>
<td>-8.59%</td>
<td>6.68%</td>
<td>-4.08%</td>
<td>-2.61%</td>
<td>2.83%</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>-3.01%</td>
<td>5.80%</td>
<td>-1.51%</td>
<td>-0.15%</td>
<td>2.05%</td>
</tr>
</tbody>
</table>
2.6 Capital Flows in Default Episodes

I took data on default episodes from Benjamin and Wright (2009) and Borensztein and Panizza (2009). (I haven’t checked whether an updated dataset exists.) I define a country-year as a default episode if one of the two papers defines it as a default episode. There are 122 default episodes in our dataset of developing countries from 1970 to 2010. Just for a first pass, I calculated the mean and median of capital flows and growth for the year of default (rows 1 and 2), and the default year and the following year.

It is clear that reserves decline in the default episodes, but we don’t see a pattern of public outflows in those episodes from our dataset. \( t \) is the year of default.

<table>
<thead>
<tr>
<th>( t )</th>
<th>Total Flow</th>
<th>Private Flow</th>
<th>Public Flow</th>
<th>Non-official Public Flow</th>
<th>Reserve Accumulation</th>
<th>Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-4.96%</td>
<td>1.01%</td>
<td>-4.66%</td>
<td>-0.29%</td>
<td>-5.30%</td>
<td>-10.31%</td>
</tr>
<tr>
<td>Median</td>
<td>-4.65%</td>
<td>-0.34%</td>
<td>-4.33%</td>
<td>-0.11%</td>
<td>-3.49%</td>
<td>-2.95%</td>
</tr>
<tr>
<td>( t+1 ) Mean</td>
<td>-4.48%</td>
<td>1.15%</td>
<td>-4.91%</td>
<td>-1.18%</td>
<td>-4.62%</td>
<td>-3.40%</td>
</tr>
<tr>
<td>Median</td>
<td>-3.95%</td>
<td>0.71%</td>
<td>-2.97%</td>
<td>-0.68%</td>
<td>-2.78%</td>
<td>1.00%</td>
</tr>
<tr>
<td>( t+2 ) Mean</td>
<td>-4.49%</td>
<td>0.87%</td>
<td>-5.05%</td>
<td>-1.22%</td>
<td>-2.77%</td>
<td>2.31%</td>
</tr>
<tr>
<td>Median</td>
<td>-3.37%</td>
<td>0.79%</td>
<td>-4.13%</td>
<td>-1.13%</td>
<td>-2.33%</td>
<td>3.34%</td>
</tr>
<tr>
<td>( t+3 ) Mean</td>
<td>-4.86%</td>
<td>0.11%</td>
<td>-4.71%</td>
<td>-0.10%</td>
<td>-4.03%</td>
<td>0.17%</td>
</tr>
<tr>
<td>Median</td>
<td>-4.22%</td>
<td>-0.53%</td>
<td>-4.18%</td>
<td>-0.43%</td>
<td>-1.51%</td>
<td>2.88%</td>
</tr>
<tr>
<td>( t+4 ) Mean</td>
<td>-4.07%</td>
<td>0.98%</td>
<td>-5.36%</td>
<td>0.27%</td>
<td>-3.92%</td>
<td>-0.14%</td>
</tr>
<tr>
<td>Median</td>
<td>-3.96%</td>
<td>1.92%</td>
<td>-4.53%</td>
<td>-0.86%</td>
<td>-2.17%</td>
<td>4.94%</td>
</tr>
</tbody>
</table>

3 A Two-Period Model

3.1 Timing

The model economy is a small open endowment economy. The income shock \( y \) has the probability distribution function \( f(y) \) with support \( [y_L, y_H] \). The income shocks are i.i.d. across time. There are three types of agents: a continuum of identical households, a sovereign government, and foreign lenders. The government is benevolent and maximizes the households’ welfare.

The timing is as follows. At the beginning of the first period, the income shock \( y_1 \) is realized. The country starts with zero private asset and zero public asset, i.e., \( B_1 = 0 \) and \( A_1 = 0 \), respectively. Positive values of \( A \) and \( B \) denote assets, while negative values denote debt. The government observes the first period’s income and decides government asset/bonds \( (A_2) \) and the income tax rate \( (\tau_1) \) to finance a fixed government expenditure \( G_1 \). Then the households
choose their consumption \((c_1)\) and bonds \((b_2)\), taking as given the country’s aggregate asset level \((A_2 + B_2)\) across the public sector and the private sector. In equilibrium, the household’s choice of \(b_2\) and aggregate private sector debt \(B_2\) coincide.

(JING’S QUESTION: DOES THE TIMING OF THE DECISION MATTER? My intuition is not, since there is no other information released between the government’s and households’ debt decision.)

In the second period, after the income shock \(y_2\) is realized, the government decides whether to enforce the repayment of outstanding aggregate foreign debt \(A_2 + B_2\). If the government enforces debt contracts \((D = 0)\), the government repays its debt \((A)\) and chooses the income tax rate \(\tau^R_2\) to finance the debt repayment and government expenditure \((G_2)\). Then, households also repay their debt and consume \(c^R_2\). If the government decides to default \((D = 1)\), the government does not repay \(A\) if it is negative, and decides the income tax rate \(\tau^D_2\) to finance \(G_2\). However, if \(A\) is positive, the government can keep its saving (reserves). Then households do not to repay their debt either, but they suffers an income loss due to sovereign default. The households’ consumption is given by \(c^D_2\).

### Period 1

\[
y_1 \text{ is realized } \rightarrow \text{ Government chooses } A_2 \text{ and } \tau_1 \rightarrow \text{ Households choose } c_1 \text{ and } b_2
\]

### Period 2

\[
y_2 \text{ is realized } \rightarrow \text{ Government decides } D \text{ and } \tau_2 \rightarrow \text{ Households get } c_2
\]

We solve the model from the end of the second period and go backward.

#### 3.2 Government’s Default Decision in Period 2

If the government decides to enforce debt contracts, the households’ consumption is given by

\[
c^R_2 = y_2(1 - \tau^R_2) + B_2.
\]

If the government decides to default, the households’ consumption is

\[
c^D_2 = y_2(1 - \kappa)(1 - \tau^D_2),
\]

where \(\kappa\) denotes the fraction of income lost due to default.

The government has to satisfy its budget given its default choice. Hence, if the government enforces debt contracts, we have

\[
y_2\tau^R_2 = G_2 - A_2.
\]
If the government defaults, we have

\[
(1 - \kappa)y_2^D = \begin{cases} 
G_2 - A_2 & \text{if } A_2 > 0 \\
G_2 & \text{if } A_2 \leq 0.
\end{cases}
\]  

(4)

Combining the last two steps together, we have the second period consumption as follows. If the government enforces repayment, we have

\[
c_2^R = y_2 + B_2 + A_2 - G_2.
\]  

(5)

If the government defaults, we have

\[
c_2^D = \begin{cases} 
(1 - \kappa)y_2 + A_2 - G_2 & \text{if } A_2 > 0 \\
(1 - \kappa)y_2 - G_2 & \text{if } A_2 \leq 0.
\end{cases}
\]  

(6)

The government will enforce debt contracts as long as \(c_2^R \geq c_2^D\). Thus, if \(A_2 > 0\), we have

\[
D(y_2, A_2, B_2) = \begin{cases} 
0 & \text{if } -B_2 \leq \kappa y_2 \\
1 & \text{if } -B_2 > \kappa y_2.
\end{cases}
\]  

(7)

Thus, if \(A_2 < 0\), we have

\[
D(y_2, A_2, B_2) = \begin{cases} 
0 & \text{if } -A_2 - B_2 \leq \kappa y_2 \\
1 & \text{if } -A_2 - B_2 > \kappa y_2.
\end{cases}
\]  

(8)

The result is intuitive because there is no other penalty for default in this simple model other than the output loss. So as long as the outstanding aggregate economy debt (public and private) exceeds the output loss, the government has incentives to default. Also, when the government has positive asset \((A_2 > 0)\), the government will default only for larger amount of private debt for any realization of \(y_2\). Alternatively, the cutoff income level \(\hat{y}(A_2, B_2)\) below which default is chosen is given by

\[
\hat{y}_2(A_2, B_2) = \begin{cases} 
-B_2/\kappa & \text{if } A_2 > 0 \\
-(A_2 + B_2)/\kappa & \text{if } A_2 \leq 0.
\end{cases}
\]  

(9)

Thus, \(\hat{y}_2\) increases with more debt \(-B_2\), i.e., the government defaults for a larger range of income shocks when the private sector debt rises. (i) When \(A_2 > 0\), \(\hat{y}_2\) is independent of \(A_2\). (ii) When \(A_2 < 0\), \(A_2\) affects the default cutoff the same way as \(B_2\). Point (ii) is general even in a multiperiod model, but I suspect that point (i) is not. Instead, with future uncertainty and incomplete financial markets, \(A_2 > 0\) should decrease the default probability.
### 3.3 Foreign Lenders

Given the default decisions, the competitive risk neutral foreign lenders break even and price the debt as follows:

\[
q(A_2, B_2) = \begin{cases} 
\frac{1}{1+r}(1 - F(-\frac{B_2}{\kappa})) & \text{if } A_2 > 0 \\
\frac{1}{1+r}(1 - F(-\frac{A_2 + B_2}{\kappa})) & \text{if } A_2 \leq 0.
\end{cases}
\] (10)

The asymmetric bond prices for positive and negative public assets are due to the assumption of one-sided commitment: the government has the option to default, while the lenders always repay. If we instead assume double-sided noncommitment, the bond prices rely only on the sum of the private and public debt. When \( A_2 \leq 0 \), we have

\[
q_1(A_2, B_2) = q_2(A_2, B_2) = q'(A_2 + B_2) = \frac{1}{(1+r)\kappa}f(-\frac{A_2 + B_2}{\kappa}) > 0.
\] (11)

When \( A_2 > 0 \), we have

\[
q_1(A_2, B_2) = 0, \quad q_2(A_2, B_2) = \frac{1}{(1+r)\kappa}f(-\frac{B_2}{\kappa}) > 0.
\] (12)

### 3.4 Households’ Decisions in Period 1

When the households choose consumption \((c_1)\) and bonds \((b_2)\), they take as given the aggregate private debt level \(B_2\) and public debt \(A_2\) and the associated bond price \(q(A_2, B_2)\). In addition, the households also take as given the default decision of the government \(D(y_2, A_2, B_2)\). The households solve the following problem.

\[
\max_{c_1, b_2} u(c_1, G_1) + 
\beta \int_{y_2} [(1 - D(y_2, A_2, B_2))u(c_2^R(y_2), G_2) + D(y_2, A_2, B_2)u(c_2^D(y_2), G_2)] f(y_2)dy_2
\] (13)

s.t.

\[
c_1 + q(A_2, B_2)b_2 = y_1(1 - \tau_1)
\] (14)

\[
c_2^R = y_2(1 - \tau_2^R) + b_2
\] (15)

\[
c_2^D = y_2(1 - \kappa)(1 - \tau_2^D)
\] (16)

The households’ first order condition is given by

\[
u_1(c_1, G_1)q(A_2, B_2) = \beta \int_{y_2}^{y_H} u_1(c_2^R(y_2), G_2) f(y_2)dy_2.
\] (17)

The hypothetical first order condition that would hold if a social planner chooses \(c_1\) and
total borrowing \( A_2 + B_2 \) is

\[
u_1(c_1, G_1) [q(A_2, B_2) + q'(A_2 + B_2)(A_2 + B_2)] = \beta \int_{\tilde{y}(A_2, B_2)} u_1(c^R_2(y_2), G_2) f(y_2) dy_2. \quad (18)
\]

The \( u_1(c_1, G_1)q_2(A_2, B_2)B_2 \) term captures an externality in that the households ignore the impact of their debt choices on the aggregate bond price. The households overestimate the benefit from additional borrowing.

From the first order condition (17), we can derive \( \frac{\partial b_2}{\partial A_2} \) as follows.

\[
F(A_2, B_2, b_2) = u_1(y_1 - G_1 - q(A_2, B_2)(A_2 + b_2), G_1) q(A_2, B_2) - \beta \int_{\tilde{y}(A_2, B_2)} u_1(y_2 + A_2 + b_2 - G_2, G_2) f(y_2) dy_2 = 0. \quad (19)
\]

When \( A_2 > 0 \), we have

\[
\frac{\partial b_2}{\partial A_2} = -\frac{\partial F}{\partial A_2} = -u_{11}q^2 - \beta \int_{\tilde{y}(A_2, B_2)} u_{11}(y_2)f(y_2) dy_2 = -1 < 0. \quad (20)
\]

When \( A_2 < 0 \), we have

\[
\frac{\partial b_2}{\partial A_2} = -u_{11}q^2 - \beta \int_{\tilde{y}(A_2, B_2)} u_{11}(y_2)f(y_2) dy_2 - qu_{11}q'(-)(A_2 + b_2) - \beta u_1(\tilde{y}_2)f(\tilde{y}_2)/\kappa
\]

\[
-\frac{u_{11}q^2 - \beta \int_{\tilde{y}(A_2, B_2)} u_{11}(y_2)f(y_2) dy_2}{-u_{11}q^2 - \beta \int_{\tilde{y}(A_2, B_2)} u_{11}(y_2)f(y_2) dy_2} = -1 + \frac{qu_{11}q'(-)(A_2 + b_2) + \beta u_1(\tilde{y}_2)f(\tilde{y}_2)/\kappa}{-u_{11}q^2 - \beta \int_{\tilde{y}(A_2, B_2)} u_{11}(y_2)f(y_2) dy_2}. \quad (21)
\]

The last term of the above equation is positive, so \( \frac{\partial b_2}{\partial A_2} > -1 \).

The results are interesting. When the government saves more, i.e., when \( A_2 \) is positive and rises, the private sector fully undoes the effect by borrowing more. When the government borrows more, i.e., when \( A_2 \) is negative and declines, it lowers the bond price that the private sector faces by increasing the default probability next period. One consequence is that the agents borrow less, but they do not fully undo the effect of increased public borrowing when \( 0 > \frac{\partial b_2}{\partial A_2} > -1 \). As a result, the total borrowing goes down. The other consequences is that the agents might increase private borrowing when \( \frac{\partial b_2}{\partial A_2} > 0 \). As a result, the total borrowing goes up substantially.

### 3.5 Government’s Saving/Borrowing Decision in Period 1

The government observes the first period’s income \( y_1 \) and decides government bonds \((D)\) and the income tax rate \( \tau_1 \) to maximize households’ welfare:

\[
\max_{\tau_1, A_2} u(c_1, G_1) + \beta \int_{y_L}^{\tilde{y}(A_2, B_2)} u(c_2^D(y_2), G_2) f(y_2) dy_2 + \beta \int_{\tilde{y}(A_2, B_2)} u(c_2^R(y_2), G_2) f(y_2) dy_2, \quad (22)
\]
s.t. (i) the government budget constraint:

\[ \tau_1 y_1 = q(A_2, B_2)A_2 + G_1 \quad (23) \]
\[ \tau_2^R y_2 = G_2 - A_2 \quad (24) \]
\[ \tau_2^D y_2(1 - \kappa) = \begin{cases} G_2, & \text{if } A_2 \leq 0, \\ G_2 - A_2, & \text{if } A_2 > 0. \end{cases} \quad (25) \]

(ii) the household’s feasibility:

\[ c_1 + q(A_2, B_2)B_2 = y_1(1 - \tau_1) \quad (26) \]
\[ c_2^R = y_2(1 - \tau_2^R) + B_2 \quad (27) \]
\[ c_2^D = y_2(1 - \kappa)(1 - \tau_2^D), \quad (28) \]

and (iii) the households’ first order condition evaluated at \( B \)

\[ u_1(c_1, G_1)q(A_2, B_2) = \beta \int \hat{u}(c_2^R(y_2), G_2) f(y_2) dy_2. \quad (29) \]

In solving this problem, the government takes as given the aggregate private debt which will be chosen as the response to the government’s choice of \( A_2 \). Denote the households’ optimal debt choice by \( B(A_2) \) and the income cutoff for default by \( \hat{y}(A_2, B(A_2)) \). Plugging in these expressions and combining the households’ and government’s constraints, we can rewrite the government’s problem as:

\[
\begin{align*}
\max_{A_2} u(c_1, G_1) & + \beta \int_{y_L}^{\hat{y}(A_2, B(A_2))} u(c_2^D(y_2), G_2) f(y_2) dy_2 + \beta \int_{\hat{y}(A_2, B(A_2))}^{y_H} u(c_2^R(y_2), G_2) f(y_2) dy_2, \\
\text{s.t.} & \\
& c_1 + q(A_2, B(A_2))B(A_2) = y_1 - q(A_2, B(A_2))A_2 - G_1 \\
& c_2^R = y_2 + A_2 - G_2 + B(A_2) \\
& c_2^D = \begin{cases} y_2(1 - \kappa) - G_2, & \text{if } A_2 \leq 0, \\ y_2(1 - \kappa) + A_2 - G_2, & \text{if } A_2 > 0. \end{cases}
\end{align*}
\]

Note that the tax rate \( \tau_1 \) will be given as a residual, once the government chooses the level of \( A_2 \).

The first order condition is given by

\[
\begin{align*}
& u_1(c_1, G_1) [q(\cdot)(1 + B'(A_2)) + (A_2 + B(A_2))(q_1(\cdot) + q_2(\cdot)B'(A_2))] \\
& = \beta \int_{\hat{y}(A_2, B(A_2))}^{y_H} u_1(c_2^R(y_2), G_2) [1 + B'(A_2)] f(y_2) dy_2 \\
& + \beta I_{A_2 > 0} \int_{y_L}^{\hat{y}(A_2, B(A_2))} u_1(c_2^D(y_2), G_2) f(y_2) dy_2.
\end{align*}
\]()
When $A_2 < 0$, which implies $q_1 = q_2 = q'$, the government’s first order condition is

$$u_1(c_1, G_1) (q + (A_2 + B(A_2)) q') (B'(A_2) + 1) = \beta \int_{y_H}^{y_H} u_1(c_2^R(y_2), G_2) f(y_2) dy_2.$$  \(35\)

The left hand side is the current utility benefit of additional public borrowing. The government internalizes the impact of its additional borrowing on the aggregate borrowing cost $(A_2 + B(A_2)) q'$ and also the response of the private borrowing to public borrowing $B'(A_2)$. The right hand side is the expected cost of additional borrowing in the form of reduced future consumption when the government enforces the contract. This brings the first order condition of private sector debt in equation (17) closer to that of the hypothetical social planner in equation (18). Since $B'(A_2) > 0$ for $A_2 < 0$, the total borrowing is different in this case from the hypothetical social planner case given the different responses of $B'$ which is bigger than one or small than one.

When $A_2 > 0$, which implies $B'(A_2) = -1$ and $q_1 = 0$, the government’s first order condition is

$$u_1(c_1, G_1) (A_2 + B(A_2)) (-q_2) = \beta \int_{y_L}^{y_L} u_1(c_2^D(y_2), G_2) f(y_2) dy_2.$$  \(36\)

The left hand side is the current utility cost of additional saving and the right hand side is the benefit of additional saving in the form of increased future consumption when the government defaults.

### 3.6 Comparative Statics

We investigate how the equilibrium $D$ changes with the output cost of default. Let us define $G$, from the government’ FOC, as

$$u_1(c_1, G_1) \left[ q(\cdot) (1 + B'(A_2)) + (A_2 + B(A_2)) (q_1(\cdot) + q_2(\cdot) B'(A_2)) \right] = \beta \int_{y_H}^{y_H} u_1(c_2^R(y_2), G_2) [1 + B'(A_2)] f(y_2) dy_2 + \beta I_{A_2 > 0} \int_{y_L}^{y_L} u_1(c_2^D(y_2), G_2) f(y_2) dy_2.$$  \(37\)

Then, using the implicit function theorem,

$$\frac{\partial D}{\partial \kappa} = -\frac{\partial G/\partial \kappa}{\partial G/\partial D}.$$  

Note that $G = -FOC$. Thus, the denominator of the above equation $\partial G/\partial D = -SOC \geq 0$. The numerator is

$$\frac{\partial G}{\partial \kappa} = \beta u_1(c_2^R(\hat{y}), \hat{G}_2) (1 + B'(D)) f(\hat{y}) (\partial \hat{y}/\partial \kappa).$$
We know that \( \frac{\partial \hat{y}}{\partial \kappa} \leq 0 \). Thus,
\[
\frac{\partial G}{\partial \kappa} \leq 0 \text{ if } B'(D) \geq -1 \\
> 0 \text{ if } B'(D) < -1
\]
Then,
\[
\frac{\partial D}{\partial \kappa} \geq 0 \text{ if } B'(D) \geq -1 \\
< 0 \text{ if } B'(D) < -1
\]
\[\text{(38)}\]

\( B'(D) < 0 \) means that the households dissave when the government increases saving. If \( B'(D) < -1 \), in response to government’s saving, the households’ dissave so much that the aggregate saving \((B + D)\) actually decreases. That is, the households’ additional borrowing is more than offsetting the government saving. Thus, equation (38) implies that, when the output cost of default gets larger, the government wants to increase saving \((D)\) only if the households’ additional borrowing is smaller than the increase in government saving. On the other hand, if \( B'(D) < -1 \), the government will decrease its saving \((D)\) in response to more severe default punishment since the government knows that aggregate saving will decrease even if it saves more.

Next, we investigate how the equilibrium choice of \( D \) change with income variability. Assume that the second period income \( y_2 \) vary within \([y^L, y^H] = [y - \sigma, y + \sigma]\). Then,
\[
\frac{\partial D}{\partial \sigma} = -\frac{\frac{\partial G}{\partial \sigma}}{\frac{\partial G}{\partial D}}
\]
The denominator is positive since \( \frac{\partial G}{\partial D} = -SOC > 0 \), and the numerator
\[
\frac{\partial G}{\partial \sigma} = -\beta u_1(c_2^R(y + \sigma), \bar{G}_2)(1 + B'(D))f(y + \sigma) \leq 0 \text{ if } B'(D) \geq -1 \\
> 0 \text{ if } B'(D) < -1
\]
Thus,
\[
\frac{\partial D}{\partial \sigma} \geq 0 \text{ if } B'(D) \geq -1 \\
< 0 \text{ if } B'(D) < -1
\]
\[\text{(39)}\]

Hence, the government responds to an increase in income uncertainty in the same way as it does to larger default penalty.

It is very important to know how the households respond to the government’s saving/borrowing decision. One can get \( B'(D) \) from the households’ first order conditions. Let us define \( F(D, B) \), from the households’ FOC as
\[
F(D, B) \equiv u_1(c_1, \bar{G}_1)q(D + B) - \beta \int_{y(D+B)}^{y^H} u_1(c_2^R(y_2), \bar{G}_2)f(y_2)dy_2 = 0
\]
Then, using the implicit function theorem,

\[ B'(D) = -\frac{\partial F/\partial D}{\partial F/\partial B}. \]

\[
\frac{\partial F}{\partial D} = -u_{11}(c_1, \bar{G}) [Bq'(D + B) + q(D) + Dq'(D)] q(D + B) \\
+ u_1(c_1, \bar{G}) q'(D + B) \\
- \beta \int_{\hat{y}(D+B)}^{\hat{y}} u_{11}(c^R_2(y_2), \bar{G}_2) f(y) dy \\
+ \beta u_1(c^R_2(\hat{y}), \bar{G}_2) f(\hat{y}) \hat{y}'(D+B).
\]

\[
\frac{\partial F}{\partial B} = -u_{11}(c_1, \bar{G}) [Bq'(D + B) + q(D + B)] q(D + B) \\
+ u_1(c_1, \bar{G}) q'(D + B) \\
- \beta \int_{\hat{y}(D+B)}^{\hat{y}} u_{11}(c^R_2(y_2), \bar{G}_2) f(y) dy \\
+ \beta u_1(c^R_2(\hat{y}), \bar{G}_2) f(\hat{y}) \hat{y}'(D+B).
\]

Note the numerator and the denominator have the same terms. Denote the same term as \( A \)

\[ A \equiv -u_{11} [Bq'(D + B)] q(D + B) + u_1 q'(D + B) - \beta \int_{\hat{y}(D+B)}^{\hat{y}} u_{11} f(y) dy + \beta u_1(c^R_2(\hat{y}), \bar{G}_2) f(\hat{y}) \hat{y}'(D+B) \]

\[ B'(D) = -\frac{A + u_{11} [q(D) + Dq'(D)] q(D + B)}{A + u_{11} [q(D + B)] q(D + B)}. \]

First, consider the case where \( D \geq 0 \) and \( B + D \geq 0 \). Then, \( B'(D) = -1 \), and the government first order condition is satisfied. In all the other cases, we cannot tell the sign of \( B'(D) \), and the government’s first order condition can hold with either sign of \( B'(D) \).
4 Model

4.1 Technology and Production

The production function is characterized by a Cobb-Douglas form that uses capital and labor as inputs:

$$y_t = e^{z_t} K_t^{1-\alpha} (\Gamma_t L_t)^\alpha. \quad (40)$$

$\alpha$ is the labor share of output. Following Aguiar and Gopinath (2007), technology consists of two parts: transitory shocks ($z_t$) and shocks to trend growth ($\Gamma_t$). $z_t$ follows an AR(1) process

$$z_t = \rho z_{t-1} + \epsilon^z_t \quad (41)$$

with $\rho < 1$, and $\epsilon^z_t \sim N(0, \sigma_z)$. The parameter $\Gamma_t$ represents the cumulative product of growth shocks. That is,

$$\Gamma_t = e^{g_t} \Gamma_{t-1} = \prod_{s=1}^{t} e^{g_t} \quad (42)$$

$$g_t = (1 - \rho_g) \mu_g + \rho_g g_{t-1} + \epsilon^g_t \quad (43)$$

with $\rho_g < 1$, and $\epsilon^g_t \sim N(0, \sigma_g)$. The parameter $\mu_g$ is the long-run mean productivity growth rate.

The law of motion for capital is

$$K_{t+1} = (1 - \delta) K_t + I_t. \quad (44)$$

Following Park (2015), gross investment $I_t$ is a CES aggregator that combines a domestic investment good $i^d_t$ and a foreign investment good $i^f_t$.

$$I_t(i^d_t, i^f_t) = [\lambda (i^d_t)^{\varepsilon_d} + (1 - \lambda) (i^f_t)^{\varepsilon_f}]^{\frac{1}{\varepsilon_f}}, \quad (45)$$

where $0 < \lambda < 1$ and $\varepsilon < 1$. The elasticity of substitution between domestic and foreign investment goods is $[\frac{1}{\varepsilon_d - 1}]$, and thus domestic and foreign investment goods are imperfect substitutes.

The foreign input $i^f_t$ must be paid in advance using working capital financing. Working capital loans $\kappa_t$ are within-period loans provided by foreign creditors. The working capital constraint can be written as:

$$\frac{\kappa_t}{1 + r} \geq p_f i^f_t, \quad (46)$$

where $r$ is the risk-free interest rate and $p_f$ is the price of imported investment goods. This working capital constraint will hold with equality at the optimum.

The price of one unit of investment good is

$$P_t = [\lambda^{\frac{1}{\varepsilon_d}} + (p_f(1 + r))^{\frac{1}{\varepsilon_f}} (1 - \lambda)^{\frac{1}{\varepsilon_f}}]^{\frac{\varepsilon_f}{\varepsilon_d - 1}} \quad (47)$$

In financial autarky, firms cannot finance foreign investment goods, thus investment and the
price of investment are given by

\[ I_t = \lambda^{\frac{1}{2}} (i_t^d) \tag{48} \]

\[ P_t = \lambda^{\frac{-1}{2}} \tag{49} \]

Firms choose employment and investment to maximize profits:

\[ \Pi_t = y_t - w_t L_t - P_t I_t - \Phi(K_t, K_{t+1}) \tag{50} \]

where \( \Phi(K_t, K_{t+1}) \) is a convex capital adjust cost function.

### 4.2 Households

The households maximize utility

\[ E_t \sum_{t=1}^{\infty} \beta^t u(c_t, L_t) \tag{51} \]

subject to

\[ w_t L_t + b_t - q_t b_{t+1} + \Pi_t = c_t + T_t \tag{52} \]

The households borrow one-period foreign debt to smooth consumption. They own firms and pay lump-sum taxes \( T_t \) to the government. In autarky, the budget constraint becomes

\[ w_t l_t + \Pi_t = c_t + T_t \tag{53} \]

### 4.3 Government’s Budget Constraint

The government has a given stream of expenditures financed by lump-sum taxes and foreign debt\[^1\]. The budget constraint for the government is given by:

\[ T_t = G_t - A_t + q_t A_{t+1}. \tag{54} \]

In financial autarky, it becomes

\[ T_t = G_t. \tag{55} \]

### 4.4 Recursive Formulation

#### 4.4.1 Timing

After observing current shocks (\( z \) and \( g \)) and the aggregate debt level \( (A + B) \) at the beginning of the normal period, the benevolent government decides whether to enforce debt contracts and chooses reserves \( (A') \) to maximize the representative household’s welfare. Then, the private sector makes decisions on \( c, b' (= B') \), \( K' \). We study the private sector’s problem first and go backward.

---

\[^1\] Later, we may consider a case in which the government provides working capital loans to firms.
4.4.2 Private Sector

Let \( s = \{ z, g \} \). The repayment value function of the household is given by:

\[
v^R(s, b, K, A, A', B') = \max_{c, L, i, d} \left[ u(c, L) + \beta E_{s'|s} \left[ (1 - D(s', K', A', B'))v^R(s', b', K', A', A'', B'') + D(s', K', A', B')v^D(s', K', A', A'') \right] \right]
\]

subject to

\[
c + PI + q(s, K', A', B')b' + \Phi(K, K') + (q(s, K', A', B')A' - A) = e^z K^{1-\alpha}(\Gamma L)^\alpha + b
\]

\[
K' = (1 - \delta)K + I
\]

\[
I = [\lambda (id)^\varepsilon + (1 - \lambda)(if)^\varepsilon]^\frac{1}{\varepsilon}, \quad \frac{K}{1 + r} = pf^f, \quad I \geq 0
\]

\[
P = [\lambda \frac{1}{1-r} + (pf(1+r))\frac{e^\varepsilon}{\varepsilon-1}(1 - \lambda)\frac{1}{1-r}]^\frac{\varepsilon-1}{\varepsilon}
\]

\[
B'' = \Psi(s', B', K', A', A'')
\]

\[
A'' = \Omega^R(s', B', K', A').
\]

\( \Psi \) and \( \Omega^R \) are the private sector’s perceived laws of motion for aggregate debt and reserves. \( D \) is the default set determined by the government.

The default value is

\[
v^D(s, K, A, A') = \max_{c, L, i, d} \left[ u(c, L) + \beta E_{s'|s} \left[ \theta v^R(s', 0, K', A', A'', B'') + (1 - \theta)v^D(s', K', A', A'') \right] \right]
\]

subject to

\[
c + PI + \Phi(K, K') + G = e^z K^{1-\alpha}(\Gamma L)^\alpha
\]

\[
K' = (1 - \delta)K + I
\]

\[
I = \lambda \frac{1}{2} id, \quad I \geq 0
\]

\[
P = \lambda^{-\frac{1}{2}}
\]

\[
B'' = \Psi(s', 0, K', A', A'')
\]

\[
A'' = \Omega^D(s', 0, K', A').
\]

4.4.3 Government

The government chooses \( A' \) to maximize the private sector’s welfare. This welfare is given by \( v^D(s, K, A, A') \) if the government chooses to default, and \( v^R(s, B, K, A', \Psi(s, B, K, A, A')) \) if the government chooses to enforce the repayment with an anticipation that the economy will borrow \( B' = \Psi(s, B, K, A, A') \) this period. Thus, the government solves the following problem:
In the beginning of each period, the government decides whether to enforce debt contracts to maximize the private sector’s welfare. This problem can be written as:

\[
\begin{align*}
\Omega^R(s, B, K, A) &= \arg \max_{A'} v^R(s, B, K, A', \Psi(s, B, K, A', A')) \\
\Omega^D(s, 0, K, A) &= \arg \max_{A'} v^D(s, K, A, A')
\end{align*}
\] (58)

(59)

(60)

where \( d = 1 \) indicates default and \( d = 0 \) indicates repayment. If the repayment welfare \( v^R \) is greater than the default welfare \( v^D \), then the government enforces the repayment of individual debt contracts. Otherwise, the government decides to declare default. Our assumption that national governments make default choices highlights default risk, driven by national governments, of private debt contracts. The governments can impose exchange or capital controls to prevent private agents from repaying their debt. In solving this problem, the government takes as given the aggregate private debt which will be chosen as the response to the government’s choice of \( D \) and \( A' \).

4.4.4 Foreign Lenders

Foreign lenders are risk neutral. They operate in competitive international financial markets and have the opportunity cost of funds at the risk-free interest rate \( r \). They thus have to break even for each debt contract. Since the government’s default decisions are based on aggregate debt, the bond price schedule also depends on aggregate debt. The zero profit condition gives rise to the bond price schedule:

\[
q(s, K', A', B') = \frac{E_{s'|s}(1 - D(s', K', A', B'))}{1 + r}.
\] (61)

Note that \( q \) is increasing in \( A' \), implying that government saving reduces private credit costs. Why is the