Introduction

- Development gap $h$ and $A$
- Getting under the hood of the Aggregate production function
- Key issues: reallocation and transmission of knowledge.
- Two sides of the same coin?

Reallocation

- Quantitatively important
- Contributes to productivity and diffusion of new ideas
- Recent contributions: barriers to reallocation very costly
- Results are very sensitive to assumptions about returns to scale or demand elasticity

Returns to scale and knowledge transmission

- Fixed factors vs. replication
- What is fixed or not may depend on incentives for knowledge transmission
- Develop a deeper theory of replication/knowledge transmission

Roadmap

1. Background: example on reallocation and returns to scale
2. Existing theories of heterogeneity and reallocation
3. Theory of learning and replication of knowledge
4. Policy experiment - sensitive to incentives for knowledge accumulation
5. Links to reallocation data
6. Further developments of the theory

Background

Example
Example: gains to reallocation and returns to scale
- firm $i$ has technology $q_i = z_i n_i^\alpha$
- $z_1 = 1$, $z_2 = z > 1$
  Total labor endowment = 2
- Initial situation: $n_1 = n_2 = 1$, $q = (1 + z)$
  after reallocation marginal products equalized.
- Upper bound for gains: $\alpha = 1$
  $n_1 = 0, n_2 = 2$ and $q = 2z$.
- $\alpha < 1$, then smaller gains.

Example: productivity gains

<table>
<thead>
<tr>
<th>$z$</th>
<th>1.5</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 0.5$</td>
<td>2%</td>
<td>5%</td>
<td>17%</td>
</tr>
<tr>
<td>$\alpha = 0.8$</td>
<td>7%</td>
<td>17%</td>
<td>39%</td>
</tr>
<tr>
<td>$\alpha = 1$</td>
<td>20%</td>
<td>33%</td>
<td>60%</td>
</tr>
</tbody>
</table>

- Gains depend on returns to scale and speed of reallocation.
- Maximum with CRS (case $\alpha = 1$)
- Models explicitly or implicitly make assumptions about this.

Theories of heterogeneity and reallocation

- Lucas span of control: $q = zg(f(k, n))$
- $z$ is managerial talent, a fixed non-reproducible asset.
- Equilibrium maximizes aggregate productivity
- Size distribution of firms $\Rightarrow$ talent distribution

Firm dynamics, entry and exit
- Hopenhayn and Rogerson: extend to entry/exit and firm dynamics
- Decreasing returns at firm level (CRS in the aggregate)
- perfect competition (infinitely elastic demand at firm level)
- Significant gains from reallocation.
The world production function and reallocation

- Eaton/Kortum/Melitz: exogenous draws for marginal cost:
  - constant marginal cost at product level (CRS)
  - constant elasticity demand.
- Trade liberalization: reallocation of world production.
- Potentially very large gains.

General observations: learning, replication and returns to scale

Learning, replication and returns to scale

- Constant returns to scale implicitly assumes costless replication: immediate transmission.
- If CRS, most efficient should be only supplier
- Opposite extreme: no replication possible. Difference in productivity ↔ fixed non reproducible factors.
- What is the fixed factor? If knowledge, embedded in HK / possible to teach.

Evidence on replication

Evidence on replication across locations

- Globalization
- FDI: over the last two decades:
  - exports have doubled
  - sales of affiliates increased more than seven times (Ramondo)
- Role of large chains in retail (Jarmin, Klimek and Miranda)

<table>
<thead>
<tr>
<th>Year</th>
<th>Single Location Firms</th>
<th>Large Retail Firms (&gt;100)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1948</td>
<td>70.%</td>
<td>12%</td>
</tr>
<tr>
<td>1967</td>
<td>60%</td>
<td>19%</td>
</tr>
<tr>
<td>1997</td>
<td>39%</td>
<td>37%</td>
</tr>
</tbody>
</table>

Explicit models of replication/learning

1. Non-rivalrous transmission
   - Chari-Hopenhayn: Learning by working with skilled workers
   - Franco and Filson: Theory of spinoffs
   - Ramondo: FDI – immediate transmission with a cost that depends on distance/ no competing alternatives

2. Rivalrous transmission
A model of learning and diffusion

- Solow (vintage model) meets Lucas (adjustment cost)
- Basic component: knowledge capital pair \((z,k)\) : \(z\) is knowledge embodied in this \(k\)
- Production technology \(zf(k,n)\), \(CRS\)
- \(f(k,n) = \min(k,n)\)

Technology for replication
- \(\dot{k}(z)\) has cost \(C\left(\frac{1}{k}\right)zk\), depreciation \(\delta k\).
- \(C(\cdot)\) increasing, convex.
- \(CRS\) in \(k, \dot{k}\)
- More costly to replicate better knowledge

Optimal replication

Optimal Accumulation
- \(v(z,t)\) value of one unit of \(k(z)\) at time \(t\)
- Bellman equation:
  \[(r + \delta)v(z,t) = z - w(t) + \left(\max_k v(z,t)\dot{k} - C(\dot{k})z\right) + v_2(z,t)\]
  - Optimal replication:
    - \(v(z,t) = C'(\dot{k})z\)
    - \(\dot{k}(z)/k(z)\) increasing in \(z\)

Equilibrium

Equilibrium with no new arrivals of \(z\)
- Fixed labor endowment \(N\)
- Initial distribution \(k(z)\) with highest \(\bar{z}\)
- Converge to steady state: \(k(\bar{z}) = N\)
- Complete reallocation: all resources flow to most productive

Equilibrium with new arrivals
• Constant flow $m$ of units of new knowledge $z\gamma(t)$
• $0 \leq z \leq 1 \sim F(dz)$
• $\gamma(t)$ grows at constant rate $g$.
• Heterogeneity in productivity within a cohort
• Coexistence of several cohorts

Balanced growth path

Balanced growth path
• $w(t) = w_0 \gamma(t)$
• Knowledge capital $z$ is discontinued when $w(t) > z$
  • Figure
• $\dot{k}/k$ declines overtime with growth of $w(t)$

Life cycle of new innovation
• Innovator starts with $k_0$ units of knowledge capital $z_0 \in [0, 1], \gamma(t) = 1$
• Active only if $z_0 > w(t)$.
• Replicates at declining rate
• Shut down after $s$ periods when $z_0 = e^{gs}w(t)$

Stationary distribution of knowledge capital

Stationary distribution of knowledge capital
• Normalize $\gamma(t) = 1$
• Steady state – stationary distribution $k(z)$
• entry, exit, growth and decline of $z$’s

Policy experiment

Policy experiment
• Tax on investment $tC(\dot{k}(z))z$
• Tax decreases investment and reallocation from less to more productive
• Impact: depends on importance of reallocation
• 3 scenarios: baseline, high adjustment cost, high $g$

Scenarios
Scenarios

- Baseline, high adjustment cost, high $g$

Knowledge replication, costs and obsolescence

Path of Capital Accumulation from $z = 1$

Reallocation

- Reallocation to growing $z'$s in unit time period as % of total capital/employment
- Reallocation to growing $z'$s
  - baseline  high $g$  high $c$
  - 3.4% 0.9% 1.5%
- Effect of taxing investment should be lower in the last two cases

Endogenous wage
Endogenous wage

- Direct effects and equilibrium effects (lowering $w_0 = w(t)/y(t)$)
- Technology for entry: flow input $n_0 \rightarrow$ rate of arrival $m$ of a unit $k$ of type $z\gamma(t)$, $z \sim F$
- Free entry condition: cost of innovation = expected value.
- Unique equilibrium value $w_0$.
- Duration of a new entry and replication decreases with $w_0$.
- Standard comparative statics

Impact of tax

- Higher taxes reduce the incentive to invest.
- But also lowers equilibrium wage
- Lower exit threshold, so less turnover
- Lower average productivity

Impact of tax

<table>
<thead>
<tr>
<th>$t$ = 0</th>
<th>Base case</th>
<th>high adj. cost</th>
<th>high g</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$w_0$</td>
<td>prod</td>
<td>$w_0$</td>
</tr>
<tr>
<td>$t = 0$</td>
<td>100</td>
<td>100</td>
<td>93.7</td>
</tr>
<tr>
<td>$t = 0.5$</td>
<td>$-6.5%$</td>
<td>$-2.1%$</td>
<td>$-3.5%$</td>
</tr>
<tr>
<td>$t = 1.0$</td>
<td>$-9.8%$</td>
<td>$-4.1%$</td>
<td>$-6.1%$</td>
</tr>
</tbody>
</table>

Connection to returns to scale

- Welfare effect: depends on aggregate supply elasticity to tax on $\dot{k}$
- Supply = $N(Q/N)$, and $N$ fixed
- Effect of tax on average productivity depends on how elastic is investment/replication to this tax
- High cost of adjustment or obsolescence behaves as an economy with low returns to scale (low $\alpha$).

Firms and Knowledge Capital

- Above considers only allocations. Data is based on firms/plants.
Firms as portfolios of knowledge capital. (Klette/Kortum: collections of products)

- State of the firm \((z_1, k_1, z_2, k_2, \ldots, z_n, k_n)\), new draws arrival rate \(m\).
- Firm grows or contracts. When number of \(z\)’s in operation goes to zero, consider an exit. Substituted by a draw for an outsider.
- Simple aggregation procedure - no change in behavior.

Firm’s life-cycle

![Firm's life-cycle diagram]

Firm dynamics - properties

- Gibrat’s law
- Growth declines with age
- Survival: Firms running more vintages are less likely to exit - also tend to be larger:
- Exiting firms 45% size of incumbents (US 35%)
- Rate of entry/exit: 5% (determined by \(w_0\)) - slightly less than US (7%)
- Total creation (and destruction) rates: 7% (US = 10%)
- Share of entry/exit: 30% (US=20%)
- Productivity decomposition (Bailey, Bartelsman & Haltiwanger): very similar to US data
- Counterfactual: age and exit rates.

Productivity decomposition

- Bailey, Bartelsman & Haltiwanger

<table>
<thead>
<tr>
<th>Productivity decomposition</th>
<th>model</th>
<th>data</th>
</tr>
</thead>
<tbody>
<tr>
<td>within establishments term</td>
<td>47.2</td>
<td>48</td>
</tr>
<tr>
<td>between establishments term</td>
<td>7.5</td>
<td>-8</td>
</tr>
<tr>
<td>interaction term</td>
<td>26.7</td>
<td>34</td>
</tr>
<tr>
<td>net entry</td>
<td>15.5</td>
<td>26</td>
</tr>
</tbody>
</table>
Costly adjustable knowledge and curvature

- Reduced form decreasing returns technology?
- Suppose a firm has $k(z)$ knowledge capital for different $z$’s.
- Desired expansion of capital (and employment) $\dot{k}$.
- Suppose limit exogenously firm’s expansion to $\dot{k}_{\text{max}} \in [0, \dot{k}]$.
- Look at $d\ln q/d\ln n$ for values of $\dot{k}_{\text{max}}/\dot{k}$.

Extensions: Economic Geography

- Knowledge can be replicated at different locations (e.g. retail chains)
- Evidence suggests distance matters: gravity equation
  - Ramondo: countries twice as far $\rightarrow$ 45% higher cost to FDI
  - Evidence from Walmart’s sequential location
  - Diversity of sizes of retail chains

Replication through locations

- Locations $l = 1, \ldots, L$
- For $i, j \in L$, $c_{ij}z$ is cost of creating $k_0$ units of capital at $j$ if have one unit of capital at $i$.
- $\max(0, v_j(z) - c_{ij}z)$ gives $L_i(z)$ locations to which choose to replicate from $i$.
- Conjectured properties: transfers will occur for high $z$’s and low $c_{ij}$’s.
- Model generates chains of knowledge capital.

Origin and destination of knowledge capital

- Assume cost of adjustment are lower in more dense areas.
- Implies that -all things equal- $w_0$ higher in more dense areas and more entry.
So distribution of active $z'$s will be better
Larger chances of originating knowledge capital

Final Remarks
The study of diffusion through replication of knowledge important area.
Important to understand the gains from reallocation and overall productivity.
Reduced-form returns to scale have implicit assumptions about replication.
Incentives to replicate may vary significantly across economies, time and space
Need for deeper models to understand overall process and incentives for knowledge transmission across time and space.