Dynamic Games and Contracts

Insights from a Decade of Research, and Recent Directions

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Motivating Examples

- Collusion with private information about costs, repeated auctions
  - Athey, Bagwell, Sanchirico (2004); Athey and Bagwell (2001); Athey and Bagwell (2008); Hopenhayn and Skrzypacz (2004); Harrington and Skrzypacz (2008)
- Public good problems
- Trading favors (academic department hiring, within-firm cooperation, legislatures, families) (Hauser and Hopenhayn (2008), Abdulkadiroglu and Bagwell (2005))
- Bilateral bargaining (Athey and Miller, 2007)
- Monetary authority (Athey, Atkeson, Kehoe, 2008)
- Individual dynamic inconsistency problem (Amador, Angeletos, Werning (2005)
- Relational Contracts (Levin (2003), Rayo (2007),
- Repeated procurement
- Dynamic Public Finance (Golosov, Tsyvinski, and Werning (2006) survey)

Class of Dynamic Problems

- Efficient decision depends on private information dispersed among agents (never publicly observed or audited)
- Information changes over time
  - i.i.d. v. correlated over time
  - Affected by decisions (learning by doing, experimentation, capacity)
- Transfers of money/utility (?)
  - None/wasteful (single agent with objective aligned with principal; agents anonymous; strongly symmetric equilibria)
  - Transfer utility but not money (asymmetric equilibria, trading favors)
  - Leaky bucket (inefficient gift giving, bribes with detection probability)
  - Transfer money
- Commitment?
  - Far-sighted agents with self-enforcing agreements
  - Formal contracts (participation constraints?)

General Economic Questions

- Efficiency: can it be sustained? Is it optimal for agents?
  - Cases w/o a principal, includes budget balanced transfers or perfectly efficient transfers of utility
  - Tradeoffs between efficiency in one period and efficiency in future periods (inefficient rewards and punishments)
  - What kinds of inefficiency occur, and when does it happen?
  - Collusion case: want to get most efficient firm to produce in each period, but all firms like market share.
- Transfers
  - Leaky bucket transfers v. transfers of favors/utility
  - Impact on efficiency, relaxation of incentive constraints
  - Collusion case: why use side payments if they can send you to jail? What are tradeoffs?

- Communication
  - When does it help? Hurt?
  - Collusion case: why communicate when it can send you to jail? What are tradeoffs?
- Less patient agents
  - Escape clauses and dynamics
  - Inefficiency
- Partial persistence
  - Do incentives for signaling and multi-stage deviations undermine efficiency?
  - Can we understand real-world behavior, such as intentionally triggering a price war, through the lens of these models?

Model: A Dynamic Game with Time-Varying Hidden Information

- Players $i = 1, .., I$
- Time $t = 1, .., T$ (special cases: $T = 1, T = \infty$)
- Superscript/subscript notation: given $((y_{i,t})_{t=1}^{T})_{i=1}^{I}$,

$$y_t = (y_{i,t})_{i=1}^{I}, \quad y_i = (y_{i,t})_{t=1}^{T}, \; y^t = (y_{t'})_{t'=1}^{t}.$$

- Type spaces $\Theta_{i,t} \subseteq \mathbf{R}^n$, random variables $\tilde{\theta}_{i,t}$ with realizations $\theta_{i,t}$.
- Communication amoung players: $m_{i,t} \in \mathcal{M}_{i,t}$
- Decisions $X_{i,t} \subseteq \mathbf{R}^n$.
- Transfer from player j to player i :
  $y_{j,i,t} \geq 0$, let $y_{i,t} = \sum_j y_{j,i,t} - y_{i,j,t}$.
  - Some models rule out transfers, e.g. collusion
- Period payoffs

$$\pi_{i,t}(x^t; \theta_{i,t}) + y_{i,t}$$

- History has two components:
  - Public history $h^{t-1} = (x^{t-1}, m^{t-1}, y^{t-1})$, private histories θ^{t-1}
- Timeline in period t:
  - Types realized (θ_t)
    - History potentially affects distributions: $F_t(\theta_t; x^{t-1}, \theta^{t-1})$.
  - Players communicate (m_t)
  - Players simultaneously make decisions (x_t) and send transfers (y_t)
- Note: can consider models without communication in this framework
  - Messages can be contentless
  - Athey-Bagwell (2001) show this can relax incentive constraints

Methodological Innovations

- Analyze decentralized games using mechanism design tools
  - Apply existing toolkit
  - Understand dynamic behavior as an attempt to mimic mechanism design ideal
- New mechanism design results for new environments
  - Fully modeled motivation for constraints on transfers
  - Impact of type-dependent constraints
  - New dynamic mechanism design results
  - Impact of realistic constraints (e.g.within-period ex post IC)
  - Role of budget accounts

Some Key Old Theorems

- Team mechanism: each player receives sum of other players' payoffs as a transfer
  - Ex post IC, not BB or IR
- d'Aspremont-Gerard-Varet mechanism for static mechanism design problem with transfers
  - Each agent gets expectation (over opponent types) of opponent payoffs
  - "Expected externality mechanism"
  - Budget Balanced (since transfers depend only on own type), Interim IC, NO IR, i.i.d. types
- Envelope theorem, revenue equivalence theorem
- Fudenberg, Levine and Maskin (1994) folk theorem
  - Small changes in future per-period utility mimic transfers
  - Key elements of argument
  - Average period payoffs (outside set) and hyperplane (inside set)
  - As $\delta \to 1$, length of hyperplane shrinks fast enough

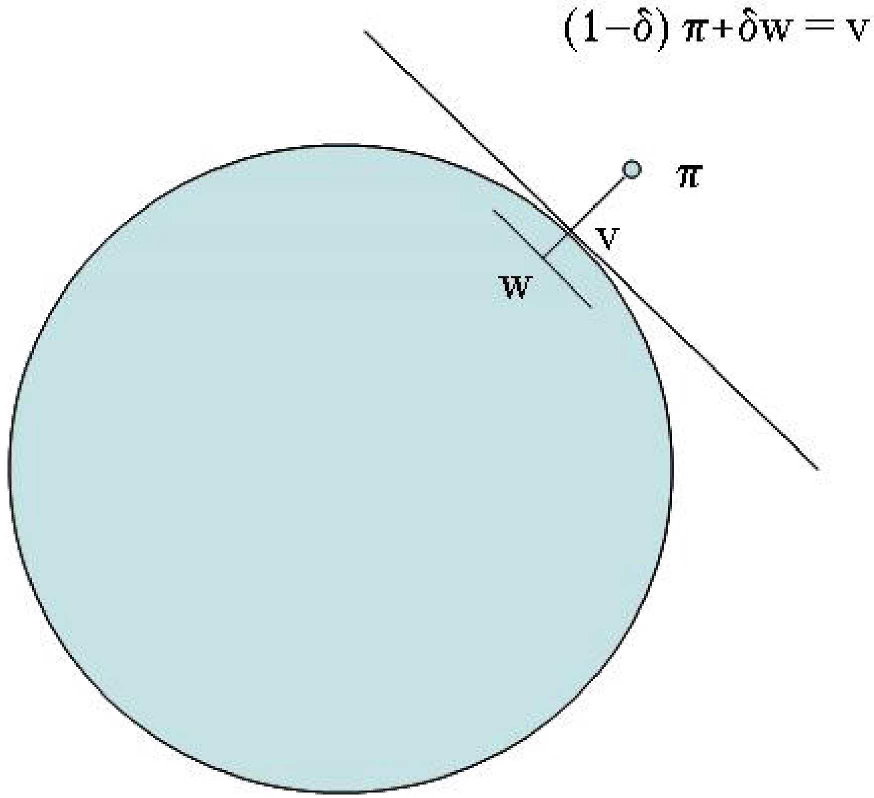


Figure 1:

FIGURE: Supporting Hyperplanes

Some Key New Theorems I: Efficiency is Possible in General Dynamic Mechanisms (Athey and Segal, 2007)

- Model: Arbitrary persistence of types; actions affect type distributions. Hidden actions affect only agent's own types.
- Theorem: Assume private values. A mechanism that gives team transfers in each period (each player receives sum of others' payoffs) is ex post incentive compatible and efficient (but not BB, IR).
- Theorem: Assume IPV. Then, there exist budget balanced transfers that implement efficiency in a bayesian IC mechanism (not IR).
  - Multi-stage deviations complicate the analysis; players' reports/actions today affect future beliefs and future transfers
  - Generalizes DGV expected externality mechanism, with adjustments for the fact that reports affect expectations in future
  - Transfers give players, in each period, *change* in expected discounted value of externalities on others, where change is induced by the arrival of new private information
  - Paid for the entire future in each pd; but these cancel out over time due to differencing

Some Key New Theorems I: Efficiency is Possible in General Dynamic Mechanisms (Athey and Segal, 2007) cont'd

- Theorem: Assume IPV, if in addition players are sufficiently patient relative to the persistence of types, then there exists a self-enforcing equilibrium in a decentralized game where players may exit the game without penalty at any stage.
  - Need patience high relative to persistence; Myerson-Sattherthwaite theorem is a counter-example

Some Key New Theorems II: Efficiency is Possible in General, Decentralized Dynamic Games

- Theorem (Escobar and Toikka, 2009): Consider a dynamic game with exogenous private information that is independent across agents and follows a Markov process (with no explicit monetary transfers). Then for patience sufficiently high relative to persistence, payoffs are approximately efficient.
  - Shows that results from (Athey and Bagwell, 2001, 2008) are general, provides general conditions for folk theorem
  - Approach is similar to review strategies from older literature; don't give a sense of what is optimal for a fixed discount factor

Some Key New Theorems III: Revenue Maximization for Principal

- Pavan, Segal and Toikka (2007)
  - General dynamic model (similar to Athey-Segal setup) where *principal* can commit to dynamic mechanism
  - Generalized envelope theorem for dynamic games
  - Generalized revenue equivalence theorem –for a given allocation rule, the change in transfer due to the arrival of new information is pinned down
  - Necessary and sufficient conditions for an allocation rule to be implementable
- Application: repeated auction where values are correlated over time; dynamic capacity constraints or learning by doing possible
  - Revenue optimal mechanism is inefficient in the first period, and "almost" efficient thereafter (agents can "post bonds," so principal can commit to efficiency with respect to future information and extract the surplus in the first period)
  - First period: agents make a payment that determines their "handicap" in future periods; allows principal to extract surplus

– Subsequent periods: use Athey-Segal (2007) transfers for handicapped allocation

Economic Insights I: With Wasteful Transfers, Efficiency May Be Possible But is Typically Not Optimal

- Example: Principal and Agent with Same Objective
  - Parent/child; monetary authority maximizes social welfare
- Example: Inability to customize dynamic play to individuals
  - Bidders use bidding agents in auctions; can exit and re-enter
- Model
  - Types affect efficient decision/allocation, evolve exogenously
  - Look for strategies that depend on public history
  - Actions that no type should take are deterred through punishment strategies; occur only off of the equilibrium path
- Result: under some conditions (monotone hazard), pooling (or intervals of pooling) is the best equilibrium for the agents
  - Colluding firms: price rigidity at monopoly price is optimal (full efficiency is the worst, equals myopic outcome)
  - Monetary authority: rigid monetary policy

– Individual with self-control problems: do not adjust consumption in response to preference shocks

– Parent/child: rigid rules rather than discretion

Economic Insights II: With Wasteful Transfers and Impatient Agents, Escape Clauses

- Examples/model cont'd...
- Problem:
  - Impatient agents are very tempted to cheat when they draw the most efficient cost types
  - May not be able to sustain cooperation
- Solution (i.i.d. types): introduce "escape clause"
- Collusion with i.i.d. types escape clause
  - Best collusive scheme: everyone charges rigid price, no matter what their cost, and share the market, but low cost types are tempted to undercut collusive price to gain the whole market
  - Two prices allowed: monopoly, and much lower price
  - There exists a cost type that is indifferent between the two (low price captures the market more often)
  - Low interval of cost types charges low price; no punishment follows

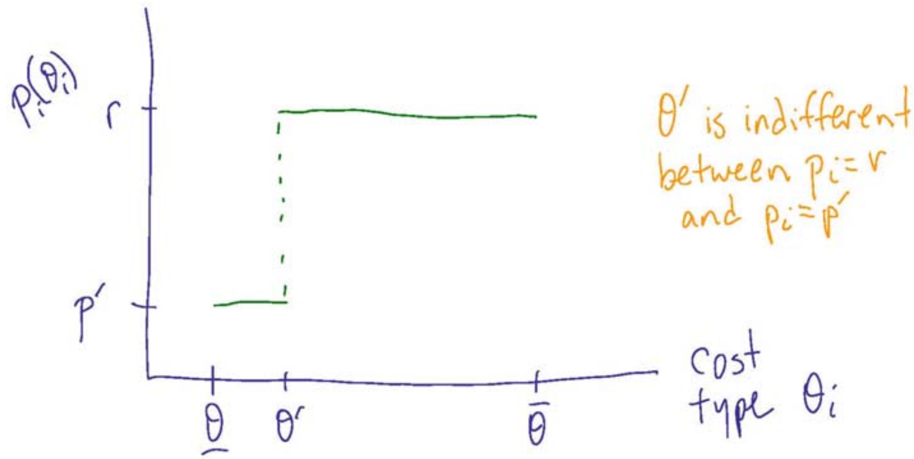


Figure 2:

- Collusion with persistent types escape clause
  - Low-cost type today will also be low-cost tomorrow
  - Initial period(s) where low-cost firm charges price below cost to signal cost type
  - In subsequent periods, receives greater market share
  - An interpretation of a price war to signal strength and bargain over market share?

Economic Insights III: Monetary Transfers + Patience Enable Efficiency, Even Without Commitment, with Interim IC

- Team mechanism: each player receives sum of other players' payoffs
  - Ex post IC, not BB
- Expected externality mechanism
  - Each player receives expectation (over opponent types) of opponent payoffs"
  - Interim IC, NO IR
- Implications for dynamic models with monetary transfers, *independent* types over time:
  - Can use expected externality mechanism in each period
  - Patience relaxes IR constraints: threaten to go to a punishment equilibrium if anyone does not take the public action or publicly deliver the transfers called for by the mechanism
  - With budget balanced transfers, no need to use continuation values for incentives on the equilibrium path: transfers do all the work of providing incentives
  - This is a special case of Athey-Segal (2007)

Economic Insights IV: Monetary Transfers with Ex Post IC or Impatience Leads to Inefficiency

- Ex post IC
  - Captures decentralized actions, incentives for spying, etc.
  - Miller (2003): no ex post IC, budget balanced transfers implies inefficiency in repeated game. Miller analyzes tradeoffs between inefficiency in one period and future punishments or money burning.
- Impatient players Levin (2003)
  - Impatience limits the scope of transfers in self-enforcing agreement
  - Inefficiency results: partial pooling is optimal
  - See also Athey and Bagwell (2001)

Economic Insights V: No Monetary Transfers, but Favor Trading, Leads to Efficiency

- Asymmetric Collusion example
- Setup
  - 2 firms produce perfect substitutes
  - Unit mass of consumers, reservation price r
  - 2 cost types: $\theta^i \in \{\theta_L, \theta_H\}$, $\Pr(\theta^i = \theta_j) = \eta_j$. Case: $\eta_L > 1/2$.
- Firms...
  - may split the market unevenly; details not imp't.
  - may not charge different prices to different consumers.
  - communicate prior to producing (see Athey and Bagwell (2001) for analysis of communication)

Summary of Ideas for Asymmetric Eq'a

- A first best scheme, always price at r
  - Eqm described by two "states"
  - Each period, announce types
  - State x: low cost firm serves market, but firm 2 serves most of market if firms have same cost
    - If (H, L), switch to state y, oth. return to x
  - State y: low cost firm serves market, but firm 1 serves most of market if firms have same cost
    - If (L, H), switch to state x, oth. return to y
- Paper: shows that first-best scheme can work if patient enough that diff. betw. x and y provides suff. incentives; if less patient shows similar schemes with partial prod. eff. are optimal.

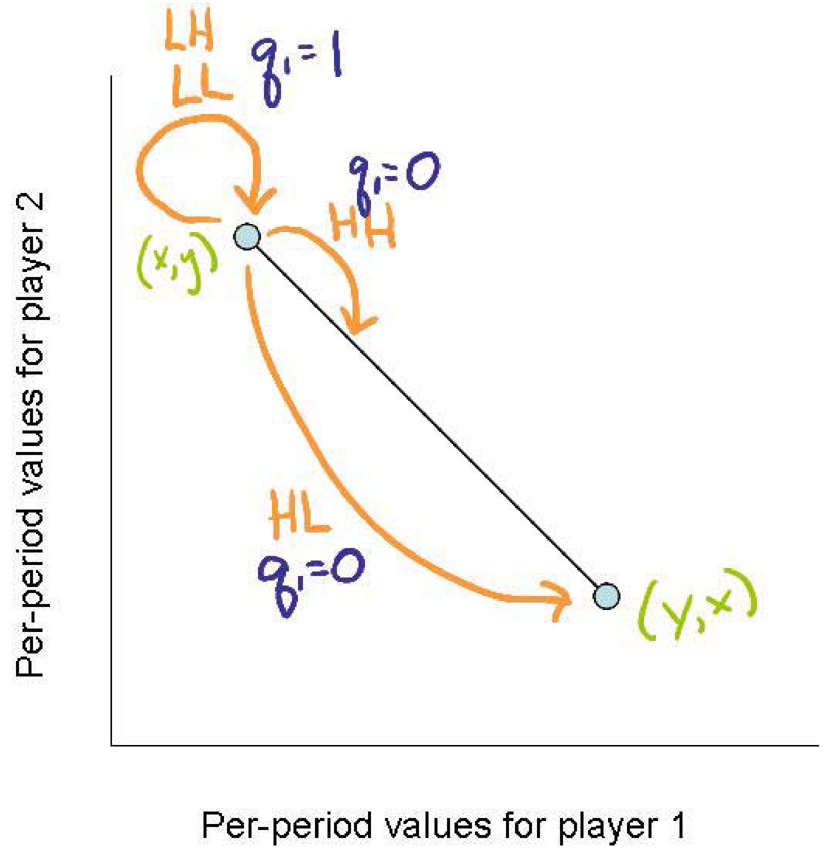


Figure 3:

Illustration of First-Best equilibrium

Economic Insights VI: Favor Trading, Communication and Bribes

- Bribes/leaky bucket
  - Favor trading can be exactly or approximately efficient
  - Always combine some favor trading with inefficient bribes
  - Bribes help for less patient firms who cannot sustain efficient, highly asymmetric equilibria
- Communication
  - Communication has a role to coordinate on market divisions
  - Sometimes communication tightens incentive constraints, because an agent who knows they will get no market share is tempted to deviate and quit

Economic Insights VII: Persistence v. Patience

- Collusion with Partially Persistent Cost Shocks (Athey and Bagwell, 2008)
  - Perfect persistence: optimal equilibrium is pooling/equal market division
  - Patience high relative to persistence: efficiency is possible through favor trading
  - Patience must dominate fact that low-cost firm today will be efficient to produce tomorrow
- Folk theorem for general games with hidden information (Toikka, 2009)

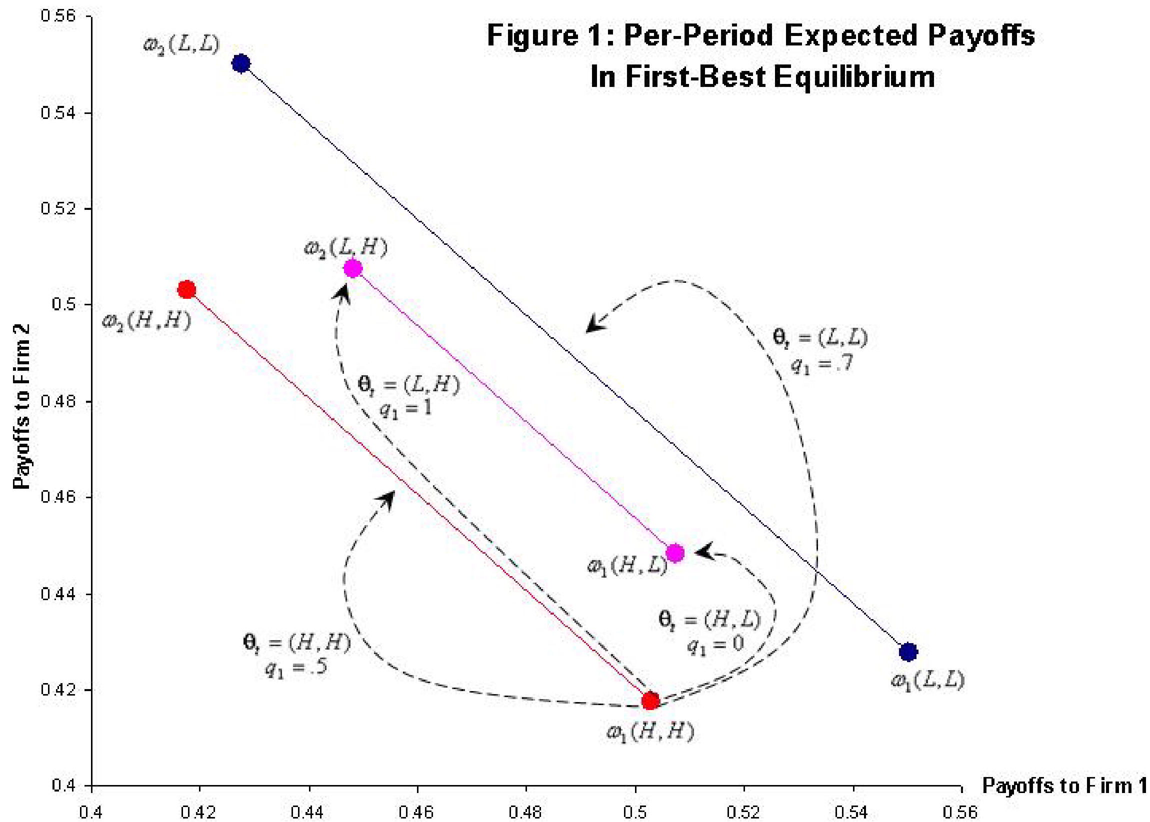


Figure 4:

Economic Insights VII: Persistence v. Patience – First Best Example with Persistent Cost Shocks

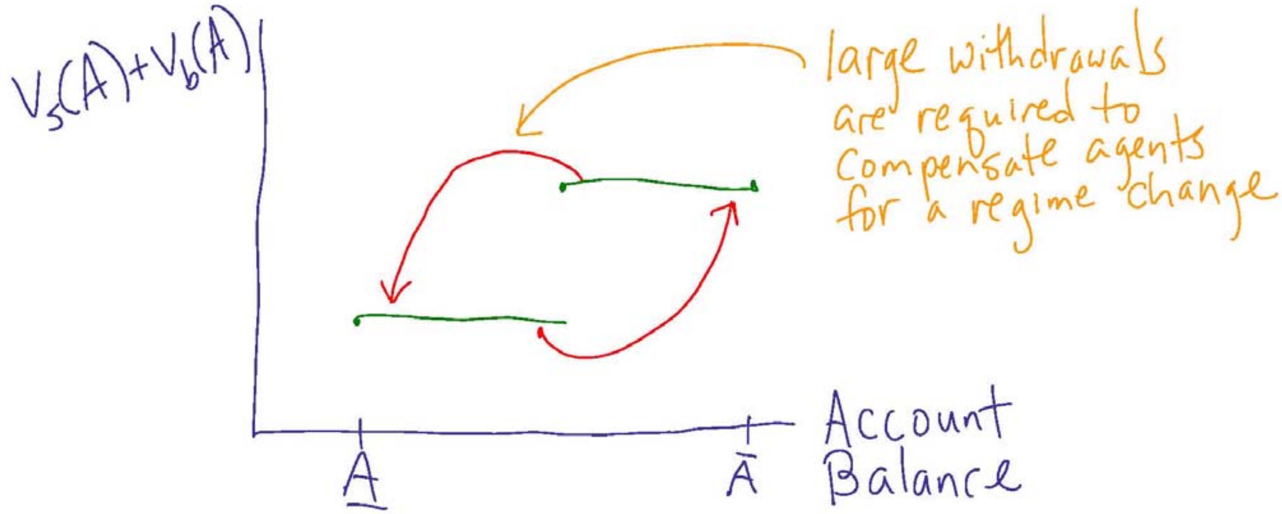
Economic Insights VIII: Ex Post IC, Budget Balance and Budget Accounts

- Recall: in static model with transfers, no ex post IC, ex post BB efficient mechanism
- Take case of bilateral trade
  - Budget can be balanced in expectation
  - Can we use a bank account to resolve budget balance issues and get efficient trade?
- Challenge: for a bounded budget account, what happens when you hit the boundaries?
  - Perfect persistence: optimal equilibrium is pooling/equal market division
  - Patience high relative to persistence: efficiency is possible through favor trading

Model of Bilateral Trade with Bounded Budget Account

- Extend Model: Efficient bilateral trade with a budget account and transfers
  - Account balance A is state variable
  - Key quantities are $v_s(A)+v_b(A)$, expected sum of discounted values, and expected sum of transfers $E[t_s(\theta; A) + t_b(\theta; A)]$.
  - Note that allocation rule and variability of transfers are the same in every period, so what varies with A is the lump sum portion of transfers
  - Net effect of withdrawal in account: change in account balance *plus* change in continuation value
  - Need to respect bounds. This affects incentives elsewhere.

- Intuition
  - At lower boundary, on average deposit a lot in order to ensure remain above the bound
  - Decreases value of being at boundary
  - If sum of values declines steeply near lower boundary, need to withdraw more just when account balance is getting low
- What happens at $A = 0$? (Efficient allocation)
  - Only deposits for all type realizations, so lump sum deposits required to preserve incentives
  - Period payoffs are low in expectation at $A = 0$
  - IR must be satisfied, so future must look sufficiently bright
    - This is not trivial even as patience grows, since near future is still near boundary and still requires deposits for disadvantaged player
- Trick: Make sum of values constant in account balance within two regions
  - Players are indifferent over account balances near boundaries
  - Two regimes: "always deposit" and "always withdraw"
  - Large transfers required to compensate for changing regime



$V_s(A)+V_b(A)$
large withdrawals are required to compensate agents for a regime change
Account Balance
$\underline{A}$
$\bar{A}$

Figure 5:

- Can handle big transfers far away from boundary

- If firms too impatient for efficiency, have an inefficient "deposit" regime at the bottom and an inefficient "withdrawal" regime at the top; can make these small

Recent Applications and Directions

- Dynamic Public Finance (Angeletos, Amador, Fahri, Golosov, Kocherlakota, Tsyvinski, Werning, etc.)
- Dynamic models of moral hazard, hidden savings (Sannikov & coauthors)
- Combining private information and private monitoring (Harrington & Skrzypacz, 2009)
- Online advertising
- Computer science literature (detail-free mechanisms, worst-case analysis, "price of anarchy," computational issues)