Optimal Wealth Taxation: Redistribution and Political Economy

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Introduction

- Wealth very unequal and skewed, on the rise?
- Industrial Revolution and Capitalism not that old
- Piketty-Saez: 20th century special due to shocks
- Increasing demands for redistribution

Q Should we tax wealth?
Q2 If so, do so progressively?
Theory?

- Theories?
  - Old Money vs. New Money
  - Zero tax!
    - **Old Money**: Chamley-Judd
    - **New Money**: Atkinson Stiglitz
  - Nonzero taxes
  - Focus: redistribution, top wealth

- **Today**: review arguments, some new ideas
Old Money
Focus on given initial wealth: *old money*

Inequality and redistribution

How to tax it?

Surprise: no tax in long run

Today: reassess these results (Werning, 2014)
Judd

- Heterogeneity
  - capitalists: save and supply capital
  - workers: supply labor, hand to mouth
- Initial wealth given
- Goal: redistribute to workers
- Taxes...
  - capital taxes on capitalists
  - rebated to workers
- Budget balance
Capitalists and Workers

\[ \sum_{t=0}^{\infty} \beta^t U(C_t) \]

\[ U(C) = \frac{C^{1-\sigma}}{1 - \sigma} \]

\[ \sum_{t=0}^{\infty} \beta^t u(c_t) \]
Capitalists and Workers

\[ \sum_{t=0}^{\infty} \beta^t U(C_t) \quad \sum_{t=0}^{\infty} \beta^t u(c_t) \]

\[ U(C) = \frac{C^{1-\sigma}}{1-\sigma} \]

\[ c_t + C_t + k_{t+1} \leq f(k_t) + (1 - \delta)k_t \]
Capitalists and Workers

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\[ c_t + C_t + k_{t+1} \leq f(k_t) + (1 - \delta)k_t \]

\[ R^*_t = f'(k_t) + 1 - \delta \]

\[ R_t = (f'(k_t) - \delta)(1 - \tau_t) + 1 \]
Capitalists and Workers

\[
\max_{\{C_t, K_{t+1}\}} \sum_{t=0}^{\infty} \beta^t U(C_t) \\
\text{s.t. } C_t + a_{t+1} = R_t a_t
\]

\[
\sum_{t=0}^{\infty} \beta^t u(c_t)
\]

\[
c_t + C_t + k_{t+1} \leq f(k_t) + (1 - \delta) k_t
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Capitalists and Workers

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s.t. \( C_t + a_{t+1} = R_t a_t \)

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\sum_{t=0}^{\infty} \beta^t u(c_t)
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\[
R_t^* = f'(k_t) + 1 - \delta
\]

\[
R_t = (f'(k_t) - \delta)(1 - \tau_t) + 1
\]
Capitalists and Workers

\[
\max_{\{C_t, K_{t+1}\}} \sum_{t=0}^{\infty} \beta^t U(C_t)
\]

subject to \( C_t + a_{t+1} = R_t a_t \)

\[
\sum_{t=0}^{\infty} \beta^t u(c_t)
\]

\[
u'(C_t) = \beta R_{t+1} U'(C_{t+1})
\]

\[
\beta^t U'(C_t) a_{t+1} \to 0
\]

\[c_t = f(k_t) - f'(k_t)k_t + T_t\]

\[c_t + C_t + k_{t+1} \leq f(k_t) + (1 - \delta)k_t\]

\[R^*_t = f'(k_t) + 1 - \delta\]

\[R_t = (f'(k_t) - \delta)(1 - \tau_t) + 1\]
Government Budget

Chamley

\[ T_t + R_t b_t = (R_t^* - R_t) k_t + b_{t+1} \]

Judd

\[ T_t = (R_t^* - R_t) k_t \]

\[ c_t = f(k_t) + (1 - \delta)k_t - R_t k_t \]
\[
\max_{C_{-1}, \{c_t, C_t, k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t (u(c_t) + \gamma U(C_t)) \\
\text{s.t. } \beta U'(C_t)(C_t + k_{t+1}) = U'(C_{t-1})k_t \\
c_t + C_t + k_{t+1} = f(k_t) + (1 - \delta)k_t \\
\beta^t U'(C_t)k_{t+1} \rightarrow 0.
\]
\[
\max_{C_{-1}, \{c_t, C_t, k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t (u(c_t) + \gamma U(C_t))
\]

s.t. \[\beta U'(C_t)(C_t + k_{t+1}) = U'(C_{t-1})k_t\]

\[c_t + C_t + k_{t+1} = f(k_t) + (1 - \delta)k_t\]

\[\beta^t U'(C_t)k_{t+1} \to 0.\]

\[
\mu_0 = 0
\]

\[\lambda_t = u'(c_t)\]

\[\mu_{t+1} = \mu_t \left( \frac{\sigma - 1}{\sigma \kappa_{t+1}} + 1 \right) + \frac{1}{\beta \sigma \kappa_{t+1} \nu_t} (1 - \gamma \nu_t)\]

\[\frac{u'(c_{t+1})}{u'(c_t)}(f'(k_{t+1}) + 1 - \delta) = \frac{1}{\beta} + \nu_t(\mu_{t+1} - \mu_t)\]

\[\kappa_t = k_t/C_{t-1}\]

\[\nu_t \equiv U'(C_t)/u'(c_t)\]
\[
\max_{C_{-1}, \{c_t, C_t, k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t (u(c_t) + \gamma U(C_t)) \\
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\beta^t U'(C_t)k_{t+1} \to 0.
\]

\[\begin{align*}
\mu_0 &= 0 \\
\lambda_t &= u'(c_t) \\
\mu_{t+1} &= \mu_t \left( \frac{\sigma - 1}{\sigma k_{t+1}} + 1 \right) + \frac{1}{\beta \sigma k_{t+1} v_t} (1 - \gamma v_t) \\
\frac{u'(c_{t+1})}{u'(c_t)} (f'(k_{t+1}) + 1 - \delta) &= \frac{1}{\beta} + v_t (\mu_{t+1} - \mu_t)
\end{align*}\]

**Judd result**

\[\kappa_t \equiv k_t/C_{t-1} \quad v_t \equiv U'(C_t)/u'(c_t)\]
Theorem. Assume allocation and multipliers converge, then the tax on capital is zero.

Two questions...
- convergence of multipliers?
- convergence of allocation?
Log Utility

\[ C_t = (1 - s)R_t k_t, \]
\[ k_{t+1} = sR_t k_t. \]
Log Utility

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\[
\max \sum_{t=0}^{\infty} \beta^t u(c_t)
\]

\[ c_t + \frac{1}{s}k_{t+1} = f(k_{t+1}) + (1 - \delta)k_t \]
Log Utility

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\[ c_t + \frac{1}{s} k_{t+1} = f(k_{t+1}) + (1 - \delta)k_t \]

\[ R^* = \frac{1}{\beta s} \]
\[ R = 1/s \]
Log Utility

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\[ \frac{R^*}{R} = \frac{1}{\beta} > 1 \]
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\[ R^* = \frac{1}{\beta s} \]
\[ R = 1/s \]

\[ \frac{R^*}{R} = \frac{1}{\beta} > 1 \] high tax!
Log Utility

- Solution...
  - time consistent
  - converges to steady state at fast rate

- What went wrong with Judd’s Theorem?
  Multipliers diverge

- Lansing (1999): “log is knife-edge case”
\( \sigma > 1 \) Case

\[
\mu_0 = 0
\]

\[
\mu_{t+1} = \mu_t \left( \frac{\sigma - 1}{\sigma \kappa_{t+1}} + 1 \right) + \frac{1}{\beta \sigma \kappa_{t+1} \nu_t} (1 - \gamma \nu_t)
\]

steady state

\[
\mu(\sigma - 1) = -\frac{1}{\beta \nu} (1 - \gamma \nu) < 0
\]
\( \sigma > 1 \) Case

\[
\mu_0 = 0
\]

\[
\mu_{t+1} = \mu_t \left( \frac{\sigma - 1}{\sigma \kappa_{t+1}} + 1 \right) + \frac{1}{\beta \sigma \kappa_{t+1} v_t} (1 - \gamma v_t)
\]

\( \gamma = 0 \)

\[
\mu(\sigma - 1) = -\frac{1}{\beta v} (1 - \gamma v) < 0
\]

\[
0 = \mu_0 \leq \mu_1 \leq \mu_2 \leq \cdots
\]
Proposition.
For $\sigma > 1$ and $\gamma = 0$ solution cannot converge to interior steady state.
$\sigma > 1$  Case
Proposition.
For $\sigma > 1$ and $\gamma = 0$ solution cannot converge to interior steady state.
Bellman Equation

\[ V(k, C_-) = \max_{(k, C_-)} \{ u(c) + \gamma U(C) + \beta V(k', C) \} \]

\[ \beta U'(C)(C + k') = U'(C_-)k \]

\[ c + C + k' = f(k) + (1 - \delta)k \]

\[ c, C, k' \geq 0 \]
\[ \sigma > 1 \quad \text{diverges} \]

\[ \sigma < 1 \quad \text{converge slowly} \]
\( \sigma > 1 \) diverges
\( \sigma < 1 \) converge slowly

both spend time close to log steady state
\( \sigma > 1 \) diverges

\( \sigma < 1 \) converge slowly

both spend time close to log steady state

large taxes in both cases rising for \( \sigma > 1 \)
Intuition

- Solution path continuous in $\sigma$
- Log case not knife edge
- Divergence...
  - increase future taxes
  - ... increases current savings
- Convergence...
  - ... lower future taxes, increase current savings
- Note: future manipulation optimal because capital is taxed
Version with heterogeneity (Werning, 2007)
Add bonds to previous model, two options...
1. workers cannot save (or tax only at top)
2. workers can save

Optimum...
- high tax rates in short run (ideal = expropriation)
- government savings from revenue

Reasonable?
Avoid first best: restrict taxation in short run
- no expropriation
- $R_1$ free

Two problems with this approach...
Avoid first best: restrict taxation in short run
- no expropriation
- $R_1$ free

Two problems with this approach...

Proposition.
1. $\sigma > 1$ and $\gamma = 0$ first best with $R_1 = 0$
2. as period shrinks first best, full expropriation
Chamley

- Alternative: maximum tax rate each period
  - Chamley: maximum tax 100% of net return
    - tax rate binds only for finite time
    - zero thereafter!
- Here:
  - any bound \( \bar{\tau} < 1 \)
  - workers cannot save
Chamley

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Proposition.
For $\sigma > 1$ and low enough $\gamma$ tax at upper bound forever
Chamley

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**Proposition.**
For $\sigma > 1$ and low enough $\gamma$, tax at upper bound forever
Chamley

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  - Any bound \( \bar{\tau} < 1 \) can
  - Workers cannot save

Proposition.
For \( \sigma > 1 \) and low enough \( \gamma \) tax at upper bound forever

\[
\bar{\tau} < 1 - \frac{K/k}{\sigma}
\]
Chamley

- Without constraints...
  - large taxation
  - asset accumulation
  - realistic?

- With constraints...
  - positive steady state taxes
General Savings

\[ k_{t+1} = S(R_t k_t; R_{t+1}, R_{t+2}, \ldots) \]
General Savings

\[ k_{t+1} = S(R_t k_t; R_{t+1}, R_{t+2}, \ldots) \]

\[
\max \sum_{t=0}^{\infty} \beta^t (u(c_t) + \gamma U(C_t))
\]

s.t. \[ c_t = f(k_t) + (1 - \delta)k_t - R_t k_t \]
\[ C_t = R_t k_t - S_t \]
\[ k_{t+1} = S_t. \]
steady state

\[ k = S(kR; R, R, \ldots) \]
steady state

\[ k = S(kR; R, R, \ldots) \]

\[ \left( \epsilon_{S,\tau} \equiv \frac{R_{\tau}}{S} \frac{\partial S}{\partial R_{\tau}} \right) \]

\[
\frac{R^*}{R} - 1 = \left( \frac{1}{\beta} - RS_I \right) \frac{1}{RS_I + \sum_{\tau=1}^{\infty} \beta^{-\tau} \epsilon_{S,\tau}}
\]
steady state

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\]

with CRRA \[ \epsilon_{S,\tau} = -\frac{\sigma - 1}{\sigma} \frac{1 - \beta}{\beta} \beta^\tau \] sum diverges!
steady state

\[ k = S(kR; R, R, \ldots) \]

\[ \left( \epsilon_{S,\tau} \equiv \frac{R_{\tau}}{S} \frac{\partial S}{\partial R_{\tau}} \right) \]

\[ \frac{R^*}{R} - 1 = \left( \frac{1}{\beta} - RS_I \right) \frac{1}{RS_I + \sum_{\tau=1}^{\infty} \beta^{-\tau} \epsilon_{S,\tau}} \]

with CRRA \( \epsilon_{S,\tau} = -\frac{\sigma - 1}{\sigma} \frac{1 - \beta}{\beta} \beta^{\tau} \) sum diverges!

- Chamley-Judd
  - does not assume infinite response of long run capital to permanent change in R (Koopmans/Uzawa)
  - infinite elasticity of NPV savings response to very distant R
Chamley-Judd

- Judd...
  - converges to zero slowly
  - may not converge to zero: constant or increasing tax rates
- Chamley...
  - high initial taxation and asset accumulation
  - binding top tax rate: long time or forever
- Case for zero taxation?
- Time inconsistency
Judd applied to Piketty

- linear technology: R and W
- W grows at rate $G = 1 + g$
- capitalists: log utility, constant savings

Assume

- $sR \geq 1$
- workers: log utility

How does tax vary with R and G?
If $\beta Rs = G$  tax constant rate $1 - \beta$

If $\beta Rs > G$
- tax converges to $1 - \beta$
- long run tax independent of $R, G, s$
- inequality not independent

If $\beta Rs < G$
- tax rate above $1 - \beta$
- eventually expropriates
- inequality extinguished

Overall: tax rate decreasing in $R/G$
New Money
New Money

- Non-inherited wealth, saved from labor earnings
- Should we tax it?
- New Money turns into Old Money...
  - inheritances...
  - ... same exact logic?
  - no, we can prepare for it
Weight on Future Generations?

- **Vc** = Utilitarian average, capture concern for equality (insurance behind veil?)
- Dynamic economy: with no weight, immiseration
Basic Model

- Farhi-Werning (2010)
- Parents
  \[ u(c_0) - h(n_0) + \beta u(c_1) \]
  - consume and work at \( t=0 \)
  - child consumes at \( t=1 \)
- Private information: productivity
- Observable
  - output and bequests (or consumption)
- Best tax systems? Trade-off
  - equality of opportunity for newborns
  - parents incentives
No Weight on Kids

- No weight on kids
- Atkinson-Stiglitz: no tax on bequests!
  - separable preferences
  - nonlinear tax on earnings
- Interpretation
  - nonlinear tax must be optimized
  - disagreements?
Weight on Kids

- Welfare function
  \[ W = \mathbb{E}[v_0 + \alpha v_1] \]
  \[ = \mathbb{E}[u(c_0) + (\beta + \alpha)u(c_1)] \]

**Proposition.**
If \( \alpha > 0 \) subsidy that falls with bequest

- Progressive tax...
  - marginal tax increasing
  - convex tax schedule \( T(b) \)
Intuition...

1. Pigouvian subsidy: but decreasing marginal utility, decreasing subsidy

2. Progressive tax creates mean reversion, lowers inequality, raising welfare
Two Properties

- Two properties of optimum...
  - negative marginal tax
  - progressive
- Negative shadow taxes in reality? Yes
  - education and other investments in kids
  - no negative bequests allowed
Positive Taxes

- Assume marginal tax rates restricted arbitrarily to being nonnegative
- What is the optimum? Zero taxation? No.

**Proposition.** Suppose production $F(K)$ is strictly concave nonnegative and increasing above some level.

- Intuition: endowment case ($F$ infinitely concave)
Two Perspectives

- Something missing in previous model
- Can we get a positive tax?

- Two perspectives...
  - For children, inheritances pure luck
    → tax and redistribute to level playing field
  - For parents → powerful argument against estate taxation
“Consider the story of twin brothers. **Spendthrift Sam** and **Frugal Frank**. Each starts a dot-com after college and sells the business a few years later, accumulating a $10 million nest egg. **Sam then lives the high life**, enjoying expensive vacations and throwing lavish parties. **Frank lives more modestly.** He keeps his fortune invested in the economy, where it finances capital accumulation, new technologies, and economic growth. **He wants to leave most of his money to his children, grandchildren, nephews, and nieces.**

Ask yourself: **Which millionaire should pay higher taxes?** What principle of social justice says that Frank should be penalized for his frugality? None that I know of.”
Basic Model

- Farhi-Werning (2012)
- Reconcile these two perspectives
- Estate taxation as balancing act
  - incentives for altruistic parents
  - equality of opportunity for newborns
Taste Shocks

- Altruism heterogeneity
  \[(1 - \theta) \log c_0 + \theta \log c_1\]
- Same income (abstract from labor)
- Children
  \[U^c(c_1)\]
- Welfare criterion
  \[\int (\lambda_\theta U^p(c_0(\theta), c_1(\theta); \theta) + \alpha_\theta U^c(c_1(\theta))) f(\theta) d\theta\]
No Weight on Kids

- Positive constant $\alpha_\theta \geq 0$
  - exists $\lambda_\theta \rightarrow$ progressive subsidy
- Constant or decreasing $\alpha_\theta \geq 0$
  - tax rate higher at top than bottom
  - tax rate non-positive at top
\( \alpha \theta = 0 \)

**constant weights**

\( \lambda \theta \) \hspace{1cm} \( \alpha \theta \)

**\( \lambda \theta \) decreasing**

\( \alpha \theta > 0 \) 

**constant**
Rawlsian

- Rawlsian constraint
  \[ U^c(c_1(\theta)) \geq u \]
- Endogenously decreasing \[ \alpha^\theta > 0 \]

**Proposition.**
Constant \[ \lambda^\theta \] and Rawlsian on children
ban on debt, positive taxes

- Intuition
  - ban on debt, satisfy Rawlsian
  - hurt the bottom, need to undo tax at top
- Binding constraint: subsidy at bottom
- Zero weight on top kids: no subsidy
Can rationalize key features of actual estate tax policy
- ban on negative bequests
- positive tax on positive bequests
Sensitive to welfare criterion
- welfare function
- cardinal normalization of utility
Heterogenous altruism?
Political Economy
Old Money Again

- New money becomes Old Money...
  - ex ante: would not want to tax
  - ex post: temptation to tax and redistribute
- Limited commitment
  - no taxation may not be credible
  - fear of discontent leading to drastic reforms, rise of communism, Chavez?
Political Economy

- How to deal with time inconsistency from redistribution?
- What policies ex ante?
- Redistribution desire depends on inequality
- Inequality is a state variable
- **Answer**: influence it!
- Compassionate conservative?
Simple Model

- Farhi, Sleet, Werning, Yeltekin (2012)
- Two periods as before...
  \[ v_0 = u(c_0) - \theta h(n_0) + \beta u(c_1) \]
- No direct extra weight on future
- Ex post: reform unless
  \[ \int u(c_1) \geq u(\kappa \int c_1) \]
- Loss of resources \( \kappa \)
Positive Taxes

$L = \int (u(c_0) + \beta u(c_1)) + \mu \left( \int u(c_1) - u(\kappa \int c_1) \right)$
Positive Taxes

\[ L = \int (u(c_0) + \beta u(c_1)) + \mu \left( \int u(c_1) - u(\kappa \int c_1) \right) \]

\[ u'(c_0) = \beta R u'(c_1) + \mu R \left( u'(c_1) - \kappa u'(\kappa \int c_1) \right) \]
Positive Taxes

\[ L = \int (u(c_0) + \beta u(c_1)) + \mu \left( \int u(c_1) - u(\kappa \int c_1) \right) \]

\[ u'(c_0) = \beta Ru'(c_1) + \mu R \left( u'(c_1) - \kappa u'(\kappa \int c_1) \right) \]
Positive Taxes

\[ L = \int (u(c_0) + \beta u(c_1)) + \mu \left( \int u(c_1) - u(\kappa \int c_1) \right) \]

\[ u'(c_0) = \beta RU'(c_1) + \mu R \left( u'(c_1) - \kappa u'(\kappa \int c_1) \right) \]

Proposition.
Best credible policy: progressive tax, positive and negative
Intuition

- Intuition
  \[ \mathbb{E}u(c_1) \geq u(\kappa \mathbb{E}c_1) \]

- Power utility functions
  \[ \frac{C\mathbb{E}u[c_1]}{\mathbb{E}[c_1]} \geq \hat{\kappa} \]

- Extra saving from...
  - poor: reduces inequality, subsidy
  - rich: increases inequality, tax
R vs G again
Political Economy

- Farhi Werning (2014)
- Parent and child
  \[(1 - \theta) \log(c_0) + \theta \log(c_1 - e_1)\]
  \[\log(c_1)\]
- Similar to warm glow
- Credibility constraint
  \[\int \log(c_1) \geq \log \left( \int c_1 \right) - \kappa\]
- Endowment growth: \( G = 1 + g \)
Linear Taxes

Proposition. (Simple Taxes)
For low $R/G$ credibility constraint not binding and optimum has no tax. For higher $R/G$ credibility binds and tax increases in $R/G$.

\[ c_0(\theta) = (1 - \theta)I \]
\[ c_1(\theta) = \theta I \frac{R}{1 + \tau} + e_1 \]
Proposition. (Nonlinear Taxes)
Optimal taxes only depend on R and G through R/G.
Conclusions

- **Old Money**
  - commitment: may tax in long run
  - no commitment: greater problem?

- **New Money**
  - commitment: subsidize/tax inheritances, progressive
  - no commitment: progressive taxes

- **Role for r - g**
  - commitment: works other way
  - no commitment: more progressive